Eastern Macedonia and Thrace Institute of Technology

Dept. of Electrical Engineering

Study of a non-linear Circuit

Μελέτη μη-γραμμικού κυκλώματος

by

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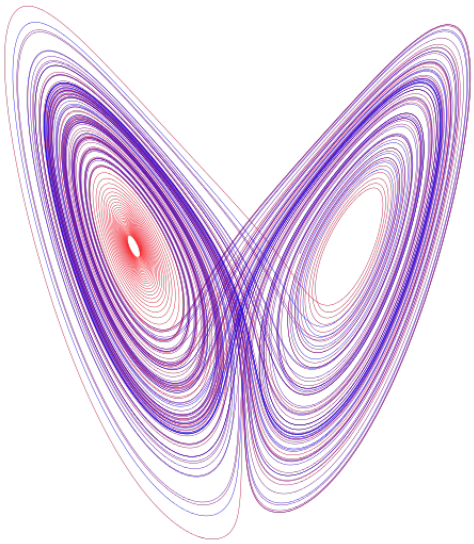

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Abstract

In this paper, the astonishing reality of chaos is probed and examined in a variety of ways. The brief history of the discovery of Chaos, the unsettling realisation that despite the fact that chaotic phenomena have been a part of the cosmos since its very formation as we've only recently discovered them, are given in order to stimulate the critical thinking faculties. An attempt to define chaotic behaviour takes place, followed by evidence of the impact that chaos has on our lives in the form of pictures, equations and thoughts. Chaotic systems are examined and are being analysed; in particular Chua's Circuit which has had a great impact on facilitating our studies of chaotic behaviour and is proven paramount to our understanding it. Chaos does indeed play an important role in our lives as it describes phenomena that everyone is familiar with and everyone encounters each day. The thesis ends with an experiment and its analysis via Auguri, a programme designed for chaotic analysis which possesses a strong Graphic User Interface element.

Keywords: Nonlinear Circuit, Chaotic Behaviour, Chaos, Chua's Circuit, Chaotic Cryptography, Synchronisation, Secure Communications.

Περίληψη

Σε αυτή την εργασία εξερευνούμε τον εκπληκτικό κόσμος του χάους. Η σύντομη ιστορία της ανακάλυψης του χάους, η ανησυχητική αντίληψη ότι παρόλο που τα χαοτικά φαινόμενα αποτελούν μέρος του σύμπαντος από την αρχή αυτού κι όμως τα ανακαλύψαμε τόσο πρόσφατα δίνονται σε μία προσπάθεια πρόκλησης της κριτικής σκέψης του ανθρώπου. Γίνεται μία προσπάθεια ορισμού του χάους, ακολουθούμενη από αποδείξεις για τον αντίκτυπο που έχει το χάος στη ζωή μας σε μορφή φωτογραφιών, εξισώσεων και σκέψεων. Εξετάζονται και αναλύονται χαοτικά συστήματα· συγκεκριμένα το κύκλωμα Chua το οποίο μας βοήθησε πολύ στις μελέτες χαοτικών συμπεριφορών και αποδείχθηκε κυρίαρχος παράγοντας της κατανόησης αυτών. Το χάος πράγματι έχει σημαντικό ρόλο στη ζωή μας καθώς περιγράφει φαινόμενα που ο καθένας παρατηρεί καθημερινά. Η πτυχιακή τελειώνει με ένα πείραμα και την ανάλυσή του μέσω ενός προγράμματος ηλεκτρονικού υπολογιστή.

Keywords: Μη-γραμμικό κύκλωμα, Χαοτική συμπεριφορά, Χάος, Κύκλωμα Chua, Χαοτική Κρυπτογράφηση, Ασφαλείς Τηλεπικοινωνίες.

1. Introduction

The nonlinear and chaotic worlds are mesmerising when one illustrates them by means of graphs and plots or when one ponders and contemplates their actual meaning and application to the real world, to our surroundings. The undeniable beauty of the Lorenz Attractor is a fine example, and with just a hint of imagination one can see the wings of a butterfly unravel from this chaotic attractor. Far from just of artistic value, chaos theory is thought to hold the key to solving a plethora of problems, from economic depressions, stock market crashes to security in communication to weather forecasting. For a long time we've failed as a species to identify chaos in nature and everyday life. However, chaos is indeed surrounding us. The trajectory of a fly's movement is a chaotic phenomenon, as is the weather and the stock market. The illusive thing that leads to an inexplicable behaviour of the physical world seems to have great minds under its spell; from the time of the great physicist Sir Isaac Newton and Pierre-Simon Laplace to the era of Albert Einstein and Neil's Bohr, a mental hunt to identify it has been taking place. Truth is, much of everyday life is described wholly or partly by chaos, and yet it wasn't until 1905 that H. Poincare, the mathematician, discovered the breathtaking field of Chaos that came to challenge the Newtonian Mechanics and broaden our minds by equipping us with new and profound ways to view the world, to think outside the box and to tackle ever more efficiently our problems.

Chaos theory, in mathematics, describes the behaviour of certain dynamical systems. A dynamic system is one whose state changes with time; a chaotic one is described by a high sensitivity to initial conditions, such that a negligible variation (by normal standards) in them can lead to a completely different outcome.

Once every few decades an invention changes the landscape of some aspects of our life. Industrial revolutions improved our everyday lives, whilst medical revolutions expanded our lifespans. The 20th century's most vital discoveries are believed to be the Chaos, Quantum and Relativity theories and yet not much interest was attributed to Chaos theory until Lorentz realised that earth's very weather behaves as a chaotic system. The reason for that might be traced back to the fact that the human brain has the tendency to linearise the information it acquires.

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2. Chaos

2.1. Defining Chaos

One can easily define in an unambiguous way what is not chaos, however, chaos itself does not seem to fall in the same category. So far there is no explicit definition of chaos, of what it entails or of its properties. In spite of this fact, scientists have indeed formed a theory of chaos, therefore we shall state that a formal definition of chaos can be thought as being correct if, and only if, it does classify essentially all mathematically well understood chaotic behaviour.

Chaos is a state of non-normality and disorder that is observed in many a dynamic system. Noise cannot be the cause of chaotic behaviour, because unlike chaos, noise trajectories span along the whole of phase space. Any system that behaves chaotically raises Attractors like the Lorenz Attractor [Fig. 0.], trajectories that twirl inside the phase space filling a certain part of it in a somewhat structural manner.

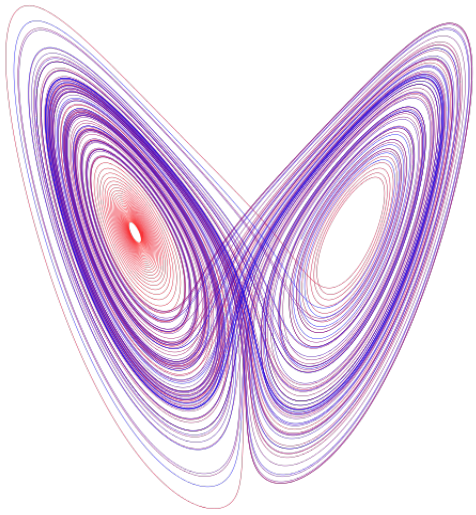


Fig. 0. Lorenz's Attractor.

Attractors that have a fractal structure like [Fig. 1.] are called Strange Attractors, and whilst many of them are chaotic, non-chaotic strange attractors also exist, with the most

common ones being the Double-scroll attractor, Hénon attractor, Rössler attractor, Tamari attractor, and of course the Lorenz attractor.

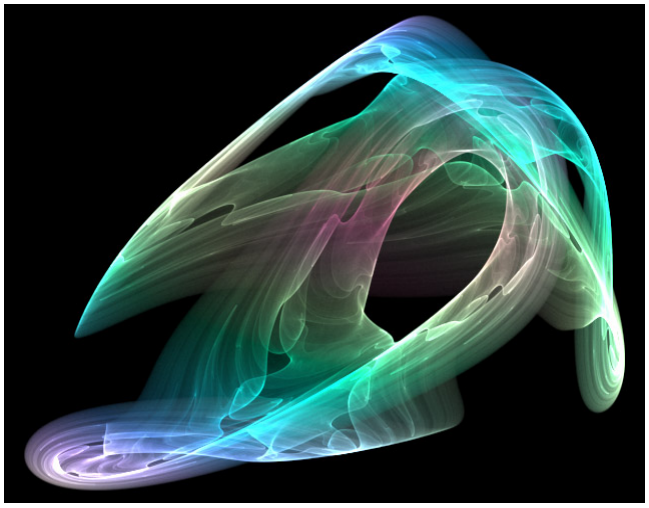


Fig. 1. Visualisation of a 3D Strange Attractor.

A great number of Degrees of Freedom cannot be the cause of chaos either. Chaos manifests as a sensitivity to initial conditions and it is an endogenous property of a non-linear system.

2.2. Theory of Chaos

Finding the Phase Space dimension provides us with the number of a system's degrees of freedom. Trajectories that do not intersect on the phase space mean there is an Embedding Dimension "m". The Correlation integrals ($C(\varepsilon)$) measure, for each phase dimension m, the possibility that two points on the attractor be found with a distance less than an arbitrary distance "r".

$$C(\varepsilon) = \lim_{N\to\infty}\left[\frac{1}{N^2}\sum_{\substack{i,j=1\\ i\neq j}}^{N}\Theta(\varepsilon - \|\vec{x}(i)-\vec{x}(j)\|)\right], \vec{x}(i)\in\mathbb{R}^m \quad (1)$$

The information loss rate is called Kolmogorov Entropy K. In a periodic signal there is no signal loss, ergo, k = 0. A random signal has an absolute entropy: K→∞, i.e. noise

The Lyapunov Exponent λ is a measure of Chaos intensity which calculates the divergence of two neighbouring trajectories that begin under slightly different initial conditions. Non positive λ coefficient indicates that the system is not sensitive to initial conditions; that is tantamount to declaring a system non-chaotic.

Let $\dot{X}$ be a vector such as Euro's exchange rate. $\dot{X}$ is a time dependent variable, hence, a time vector.

$$\dot{X} = X_1, X_2, X_3, \dots, X_N \tag{2}$$

where N is the time vector's length (i.e. the number of elements on $\dot{X}$). The impact other variables have on the system is hidden within the measured variable and according to Floris Takens' "Takens Theorem" it is possible to form virtual variables using the measured one, equivalent to the original variable, only with a slight time shift τ of its elements. The measured value is one of the system's Degrees of Freedom. In an m-dimensional space, points that correspond to the values of

$$\dot{Y}_i = \{X_i, X_{i-\tau}, X_{i-(m-1)\tau}\}, i = 1 \dots N \tag{3}$$

can be created so that if for example m = 3 then $\dot{Y}_i$'s coordinates are in a three-dimensional space

$$\left.\begin{array}{ll} m = 1 & X_i \\ m = 2 & X_{i-\tau} \\ m = 3 & X_{i-2\tau} \end{array}\right\} \dot{Y}_i = (X_i, X_{i-\tau}, X_{i-2\tau}) \tag{4}$$

For i = 10, τ = 4 $\dot{Y_{10}} = (X_{10}, X_6, X_2)$.

To figure out the optimum τ, the Average Mutual Information is invoked and $I = I(\tau)$. If a first local minimum occurs in I (mutual information) then that is the optimum τ value. To proceed to the embedding dimension one needs first invoke the correlation

dimension, which is somewhat like the fractal dimension, and draw its plot. “m” will then be [v]+1, i.e. there are [v]+1 degrees of freedom on the system.

2.3. Dynamic Systems

Dynamic Systems are those systems whose evolution from some initial condition is described by a group of equations. A system of n different equations describes mathematically a variety of dynamic systems.

$$\frac{dx}{dt} = F(x, t, c) \tag{5}$$

where x is a vector of n real time functions and is described as follows:

$$x(t) \equiv \{x_1(t), x_2(t), \dots, x_n(t)\} \tag{6}$$

or by the symbol $x \in R^n$ where R^n is the phase space. The constant c is an exogenous variable whilst c_i are the system’s control parameters. The system state on a given time on phase space is determined by a point. Said point is moving with time, creating a trajectory in the phase space according to $\boldsymbol{F}: R^n \to R^n$ vector field of (Eq. 5). Each point in the phase space corresponds to one trajectory, which is traced back to its deterministic nature of the system’s description. By extension, if the status of the system is known for one time period, it is possible to infer the past and the future by solving the integral of (Eq. 5).

2.4. Non-Chaotic Attractors

A Nonchaotic attractor is an attractor that is converging to a limit but as its Lyapunov exponent is non-positive, it is non-chaotic. On such an attractor, its trajectory on the phase space describes the evolution of the dynamic system. A fixed point that attracts the trajectories is, by definition, called an attractor and is or is not chaotic depending on its attributes. Two trajectories on nonchaotic attractors that are near each other remain as such for a long period of time for nonchaotic attractors are not sensitive to

initial conditions. By extension, the evolution of non-chaotic attractors is predictable for however long a period of time. A non-chaotic attractor can be a fixed point, a limit circle or a limit torus.

A fixed point is an attractor that reaches a state of balance. In the case of a pendulum, for example, there is friction transforming energy into heat, the speed of the pendulum gradually decreases and eventually the pendulum stops on a fixed point.

In the phase space this action is represented as shown in [Fig. 2.] in which the dots represent the initial conditions.

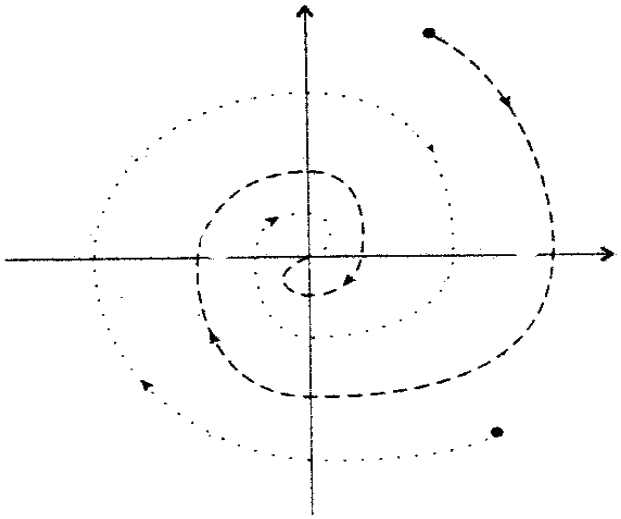


Fig. 2. Phase space of a Pendulum as it reaches a state of balance.

All the trajectories converge and eventually stop on the fixed point (0).

An example of a limit cycle is the heartbeat whilst resting. It is a periodic orbit of an isolated system. If the heart is disturbed by running, the heartbeat will return to its original trajectory after a short period of time (when the heart gets some rest), as shown in [Fig. 3.].

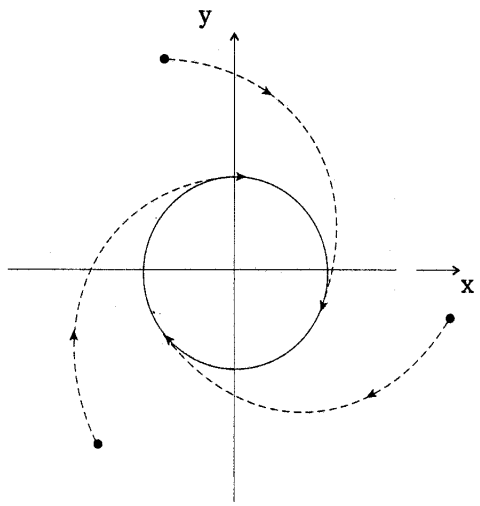


Fig. 3. Phase space of a two-dimensional system whose attractor is a limit cycle.

If an irrational fraction is formed between 2 random independent linear frequencies from a system that has *r* independent linear frequencies then the non-chaotic attractor is a limit torus. The system's movement is quasi-periodic and the trajectory on the phase space spans over the surface of a torus as shown in [Fig. 4.].

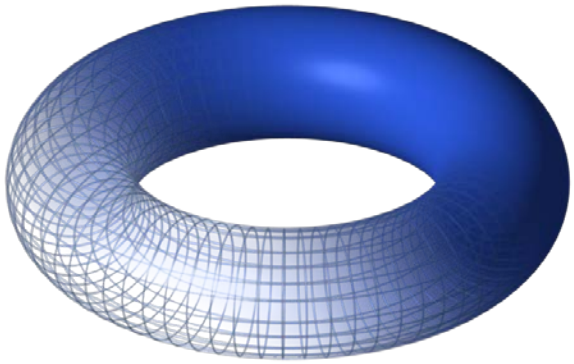


Fig. 4. A Torus attractor.

3. The Impact of Chaos in everyday life

3.1. The Chaos inside us since the moment we're born

Most people might not realise it, but chaotic behaviour might be what let them be who they are today. For an embryo, everything starts from a single cell, which is then divided into two cells, and those two cells are divided into four cell and so on and so forth, all of which are uniform, featureless. Then the cells begin to clump together and differentiate into lung cells, bone cells and so on, absent of thought, absent of coordination. This is called morphogenesis (from the Greek words "Μορφή" and "Γέννηση") meaning the birth of shape and is an example of self-organisation. A. M. Turing was the first to understand and explain self-organisation in his paper "The Chemical Basis of Morphogenesis", and he was the first to give a mathematical explanation [Eq. 7], [Eq. 8] for morphogenesis.

$$\frac{\delta a}{\delta t} = s\left(16 - a_{i,j}b_{i,j}\right) + D_a\nabla^2 a \tag{7}$$

$$\frac{\delta b}{\delta t} = s\left(a_{i,j}b_{i,j} - {}_{i,j} - \beta_{i,j}\right) + D_b\nabla^2 a \tag{8}$$

These equations are usually used in astronomy and atomic physics to describe a living process and now they describe how self-organisation can occur in biological systems and transform featureless and smooth things into something with features; without any intervention, all by itself. Complexity seems to be an inherent property of the universe.

3.2. Pattern Formation

It is truly astounding that exceptionally simple equations governing extremely simple processes result in pattern formation as a natural consequence. In the sand, for example, the steady wind blowing across creates all kinds of shapes such as [Fig. 5.].

Fig. 5. Pattern formation in the sand.

Each grain of sand is almost identical to any other and the grains have no knowledge of the shapes they become part of, and yet, the grains are self-organised into ripples, waves and dunes. Similarly to that, the chemicals seeping across an embryo might indeed cause its cells to self-organise into different organs.

In a few hours of computation, Turing got an example of dappled pattern from a type ‘a’ morphogen system [Fig. 6.].

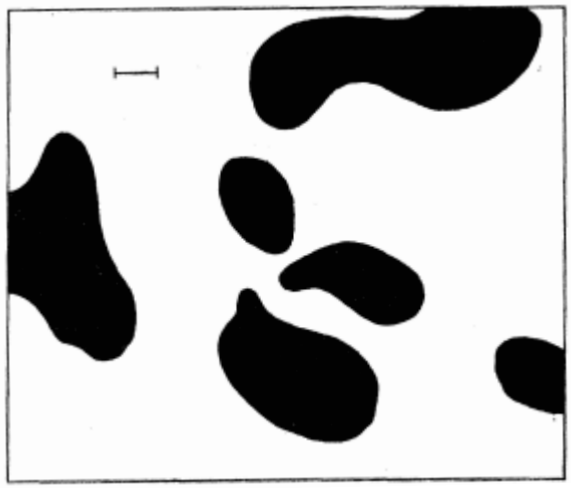


Fig. 6. Dappled pattern from a type ‘a’ morphogen system.

These strange blobs and patterns evolve on their own from a featureless chemical soup as if magic were involved. Their similarity to the markings on the skin of animals is remarkable. Another example of pattern formation from smooth, featureless chemicals is that of a variation of Belousov's chemicals.

Boris Pavlovich Belousov was a chemist who was investigating how our bodies extract energy from sugars by formulating a mixture of chemicals that mimic a part of the process of glucose absorption in the body. All of the chemicals were clear in colour, and as he poured the last one in he noticed that the chemical soup changed colour. This was nothing extraordinary, but the solution changed colour again, going back to clear. Surely two chemicals can mix and change their colour, but how can they un-mix? This seemed to violate the laws of physics, and weirder still, the solution changed colour again and again, oscillating back and forth between clear and opaque.

If a variation of Belousov's chemicals is left unstirred in a petri dish, instead of oscillating they self-organise into beautiful, breathtaking patterns and shapes out of nowhere [Fig. 7.].

Fig. 7. Self-Organisation of Belousov-Zhabotinsky chemicals.

More fascinating still, our heart's cell-coordination as the heart beats behaves exactly like the way Belousov's chemicals move as coordinated waves. Self-organisation is all over the natural world. From morphogenesis, to animal skin, to the way human organs work, to weather itself.

3.3. The Butterfly Effect

Weather is similar to Belousov's chemicals in that they both behave in very complicated ways but are based on very simple mathematical rules and a part of those rules is feedback. As with Chua's Circuit described later below, order and chaos emerge on their own from a simple system so long as there is feedback.

Order and chaos are deeply linked; all these are chaotic phenomena, and after taking this into consideration we must come to the conclusion that the natural world is profoundly unpredictable. However, this is not necessarily negative, for what makes it unpredictable is what allows it to create patterns and structure.

If we think of a camera output on a screen but the camera records that very screen which was used to show its output, then we get the classical example of a feedback loop in which a loop is created with multiple copies of what appeared on the screen. We get a picture, in a picture, in a picture which at first glance seems very predictable; strange things begin to happen as we zoom the camera, however. If, for example, we record ourselves with a lit match in that configuration, we then notice that what is being filmed bears little resemblance to what now appears on the screen, because small changes in the movement of the match became rapidly amplified as they looped from the camera to the screen and back to the camera. Even though each step of the process can be mathematically described, there is no way of predicting how the tiny changes in the flickering of the flame will end up in the image. This is precisely what we'd call "The butterfly effect", a phrase that came out of Lorenz's talk "Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?".

Yet again, the butterfly effect was all too visible on the equation [Eq. 9]

$$x_{n+1} = r x_n (1 - x_n) \tag{9}$$

that Robert May was investigating in order to model how animal populations changed over time. It seemed that seemingly negligible changes to the rates at which the animals reproduced could have enormous consequences in their overall population which in turn could go huge or incredibly small for no obvious reason. It was at that moment that the Newtonian dream that a simple equation could be used to predict how a system would behave was shattered.

3.4. The Genesis of Fractal

Fractal designs exist everywhere in nature; they are on plant leaves, animal skin and on the land. The person who identified fractal designs was Benoît B. Mandelbrot, the mathematician. He had an uncanny ability to perceive pattern and structure, form and see mathematical rules where most people could only see anarchy. He was befuddled and failed to comprehend how mathematicians were obsessed with perfect structures like a sphere or a cube when the environment is full of irregular shapes such as those of rocks, trees and clouds for which there is no proper mathematic way of describing them. Mandelbrot figured out that there is a principle that governs all those shapes, and it is called “Self-Similarity”, meaning that there is a shape which is repeated over and over again like in [Fig. 8.] where a line segment is on the bottom, that then branches into two smaller lines above which in turn branch into four smaller lines and the process is repeated.

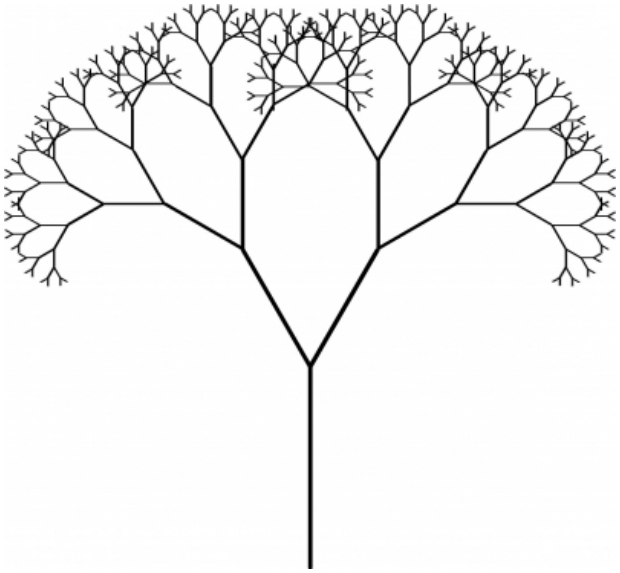
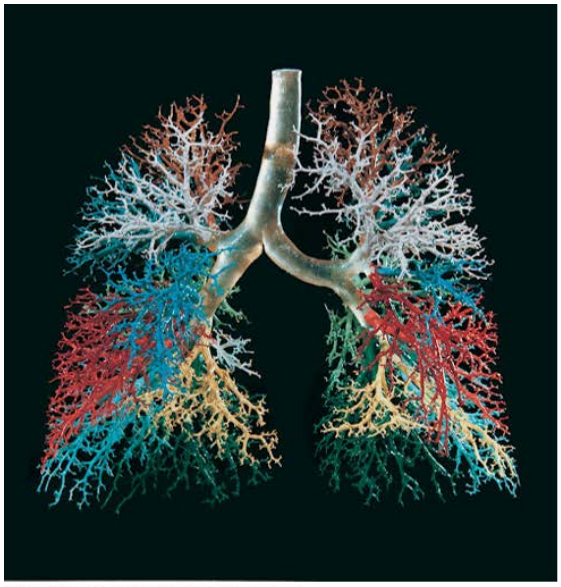


Fig. 8. Self-Similarity on a Tree.

This exactly same principle holds true for the way our lungs are formed [Fig. 9.] and the way our blood vessels are distributed throughout our bodies, for the way broccoflower is formed [Fig. 9.] and the way lightning strikes the earth [Fig. 9.], for the way rivers are formed [Fig. 9.] and zebras and tigers' skins are. Chaos and pattern, it seems, are woven into nature's most fundamental rules. Examining the lungs we see a line that branches into two lines that in turn branch into four that in turn branch again and again. The same could be said about the broccoflower which is a cone made of smaller cones that in turn are made of even smaller cones. This is indeed a new kind of geometry, named "Fractal", a term coined by Mandelbrot.



The Fractal Nature of Lungs.

The Fractal Nature of Broccoflower.

The Fractal Nature of Lightning.

The Fractal Nature of Rivers.

Fig. 9. Fractal Design examples in Nature.

Mandelbrot, however, got even further than that. In addition to identifying fractal designs in nature, he created fractal designs of his own. Having access to the incredible power of IBM's supercomputer, he was working on an astonishingly simple equation [Eq. 10] to create an infinitely complex and mind-bogglingly beautiful mathematical illustration [Fig. 10.].

$$Z \rightleftharpoons Z^2 + C \tag{10}$$

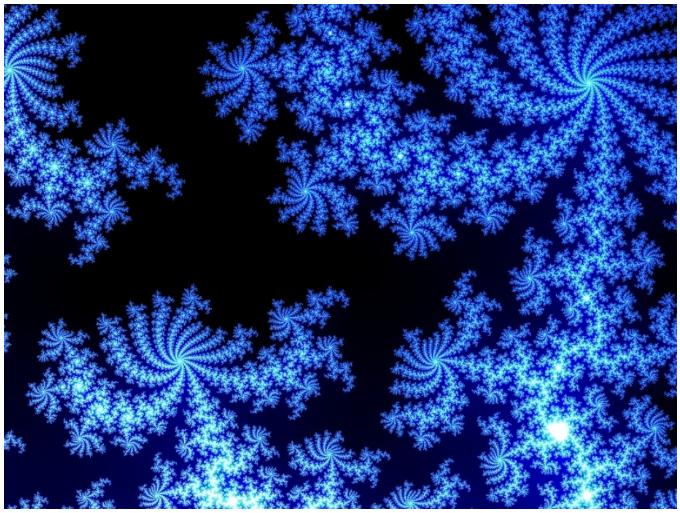


Fig. 10. Mathematical Illustration of $Z \rightleftharpoons Z^2 + C$.

This means that the equation feeds back on itself and every output becomes part of the new input as well, and it keeps happening for ever. This is why such a simple equation can result in infinite complexity.

Each shape in the picture contains an infinite number of smaller shapes, all similar to each other so that if we zoom into an edge of a shape of this picture we get [Fig. 11.].

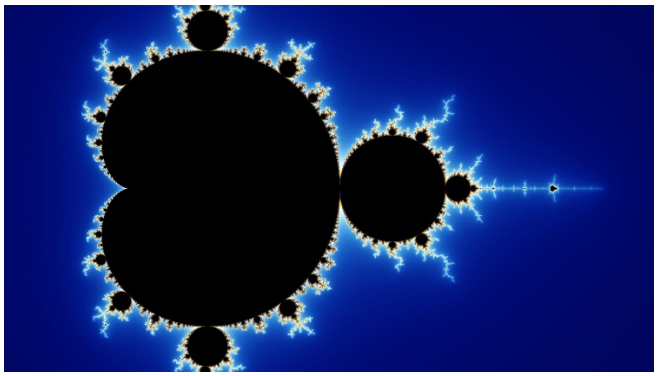


Fig. 11. Zoomed in Mathematical Illustration.

and if we zoom yet again then we get [Fig. 12.] which shows how a zoomed shape has in it parts similar to the original shape [Fig. 10.], which if we zoom even further we'll get parts similar to [Fig. 11.] again. This keeps happening ad infinitum.

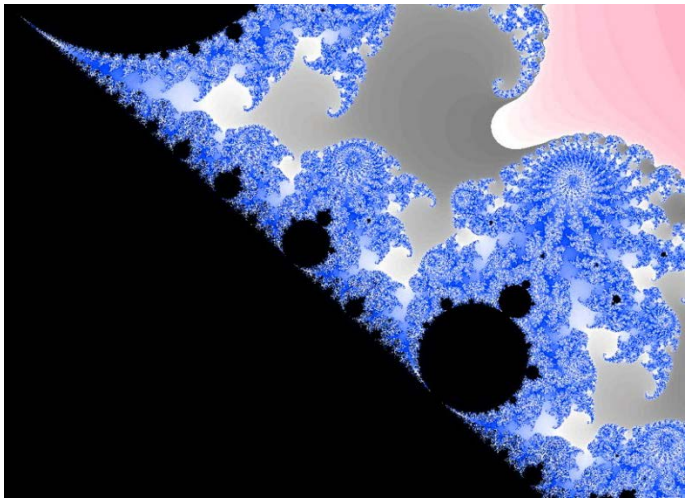


Fig. 12. Further zoomed in Mathematical Illustration.

4. Chua's Circuit

Leon O. Chua presented Chua's circuit [Fig. 13.] for the first time in Japan, 1983. This circuit is basically a chaotic oscillator that is equipped with the ability to manifest chaotic behaviour for a specific range of its parameters. It is undoubtedly the simplest electronic circuit to allow us to observe chaotic behaviour, and this very fact differentiates it from all the other electronic circuits, rendering it extraordinarily beneficial to people who want to study and do research on chaotic behaviour. It could be argued that Chua's Circuit, with its famous double scroll attractor, was a milestone for chaos research.

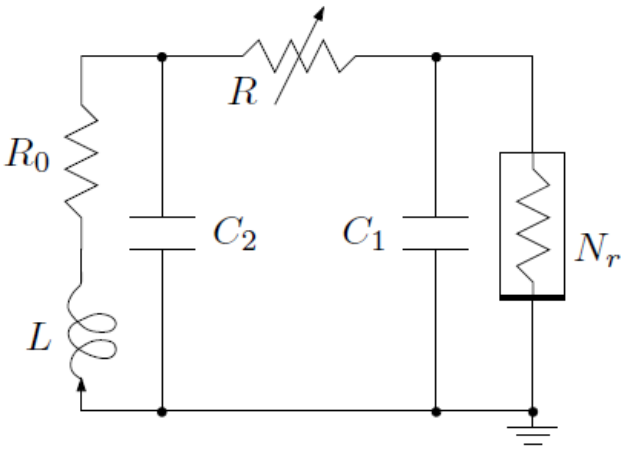


Fig. 13. Chua's Circuit.

4.1. What makes Chua's Circuit tick

In order for a circuit to manifest chaotic behaviour, three conditions must be met:

- The circuit must be comprised of one or more nonlinear elements,
- One or more locally active resistors,
- And three or more energy storage elements.

In all its simplicity, Chua's Circuit does qualify as a chaotic circuit, meeting all three conditions. It consists of three energy storage elements which are the C_1 and C_2 capacitors, the L Coil with R_0 internal Coil Resistance, an active resistor R and a nonlinear element Nr. The Nr element behaves as a nonlinear, negative impedance resistor. There are two main Nr circuits thus far. The first implementation is comprised of four linear resistors and two diodes that give it its nonlinear behaviour whilst the negative impedance is achieved via an Operational Amplifier and three resistors creating a negative impedance converter as illustrated in [Fig. 14.]. The diodes and resistors R1, R2, R3, and R4 provide it with the nonlinear behaviour and the negative impedance converter provides the negative impedance.

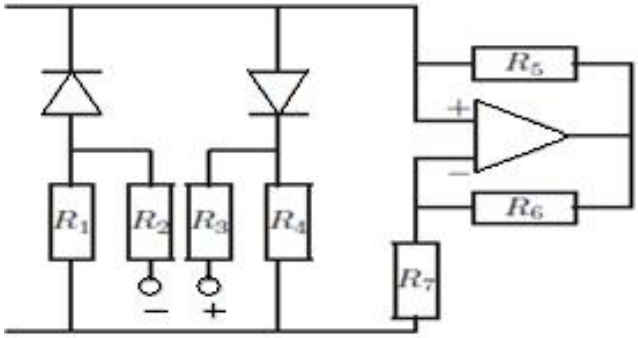


Fig. 14. Negative Impedance Converter.

The *Nr* circuit depicted in [Fig. 14.], however, does not have as good a characteristic curve as that of [Fig. 16.]. In this thesis the *Nr* circuit in [Fig. 16.] is used as *Nr*.

4.2 Chua's Circuit Equations

The three non-linear dynamic equations that describe Chua's Circuit behaviour are the following:

$$C_1\dot{V}_1 = G(v_2 - v_1) - f(v_1) \quad (11)$$

$$C_2\dot{V}_2 = G(v_1 - v_2) - i_L \quad (12)$$

$$Li_L = -v_2 - R_0 i_L \quad (13)$$

where

$$G = \frac{1}{R}, \text{ and} \quad (14)$$

$$f(v_1) = G_b v_1 + (G_a - G_b)(|v_1 + B_p| - |v_1 - B_p|) \quad (15)$$

which is the nonlinear equation that describes the nonlinear element *Nr* and is composed of three lines with G_a, G_{b+}, G_{b-} slopes as shown in [Fig. 15.].

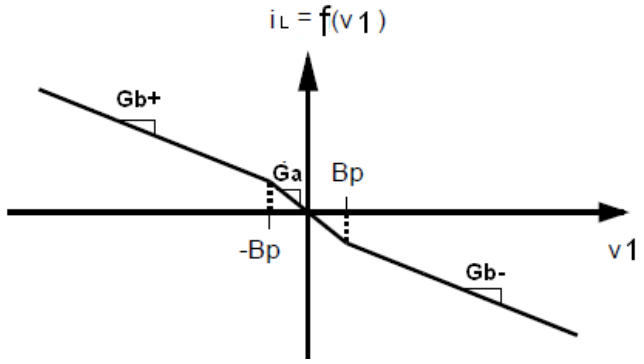


Fig. 15. V-I characteristic Curve.

5. Experiment and Computation

5.1. The Circuit

The circuit that was examined is depicted on [Fig. 16.].

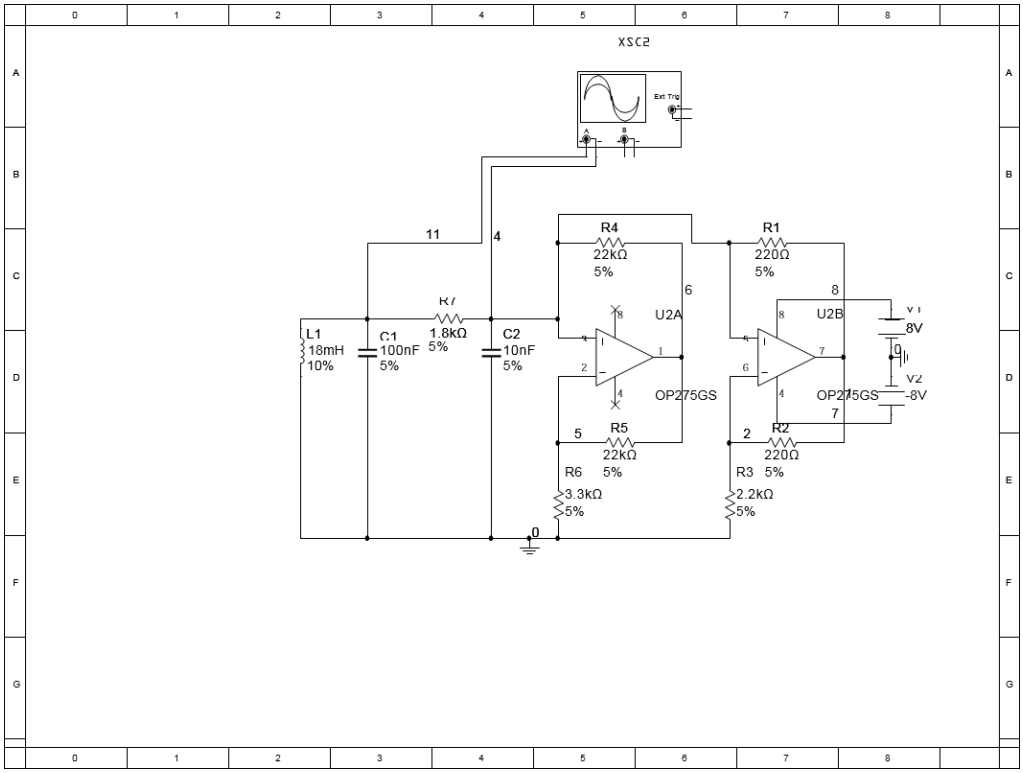


Fig. 16. Experiment's Circuit.

The Circuit is composed of one inductor, L1 at 18mH, one capacitor, C1 at 100nF, seven resistors, R1 at 220Ω, R2 at 220Ω, R3 at 2.2kΩ, R4 at 22kΩ, R5 at 22kΩ, R6 at 3.3kΩ and R7 at 1.8kΩ, two DC power sources, V1 at 8VDC and V2 at -8VDC, and finally an operational amplifier OP275GS.

The L1 and C1 is the resonant circuit which helps the system to oscillate with minimal damping. That is, the resonant circuit tries to minimise the influence on the oscillatory system that has the effect of reducing its oscillations (damping).

The R1 and R4 facilitate the positive feedback of the operational amplifier where the output of the op-amp is fed back to its positive input and an oscillation is then created.

The Operational Amplifier OP275GS is the first to feature the Butler Amplifier front end. Bipolar and JFET transistors are combined in this new front end in order to provide the accuracy and low noise performance of a bipolar transistor, but keeping the speed and quality of a JFET one.

5.2. Time Vector

In this experiment, the Auguri programme, which is described later on, analyses a time vector in order to compute various coefficients and results in general so as to determine, amongst other things, if a system is chaotic. A time vector is a collection of observations over time and it is usually represented by the variable $x(t)$ where t is time.

5.3. Auguri

Auguri is a programme that provides tools for the manipulation and analysis of data throughout the process of predictive data mining: from data capture to multivariate model design, and from the specification of solutions to these models to their comparison and storage for later use, all under a powerful and intuitive graphical user interface.

A thorough guide to the circuit's analysis with the Auguri programme follows.

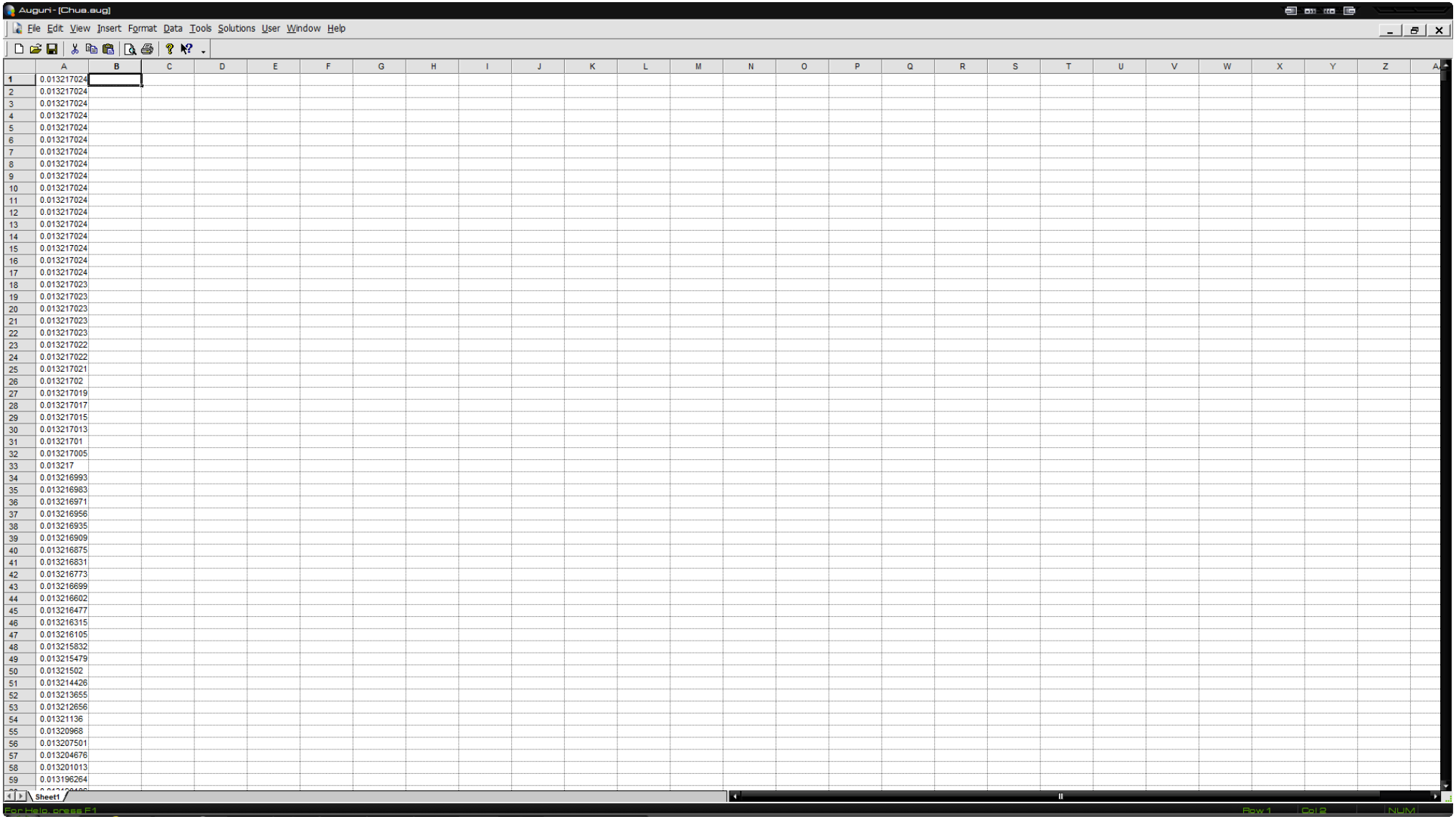


Fig. 17. Circuit's Data.

The voltage time series gathered from the circuit is opened for analysis by the Auguri programme and is displayed in the Spreadsheet within it.

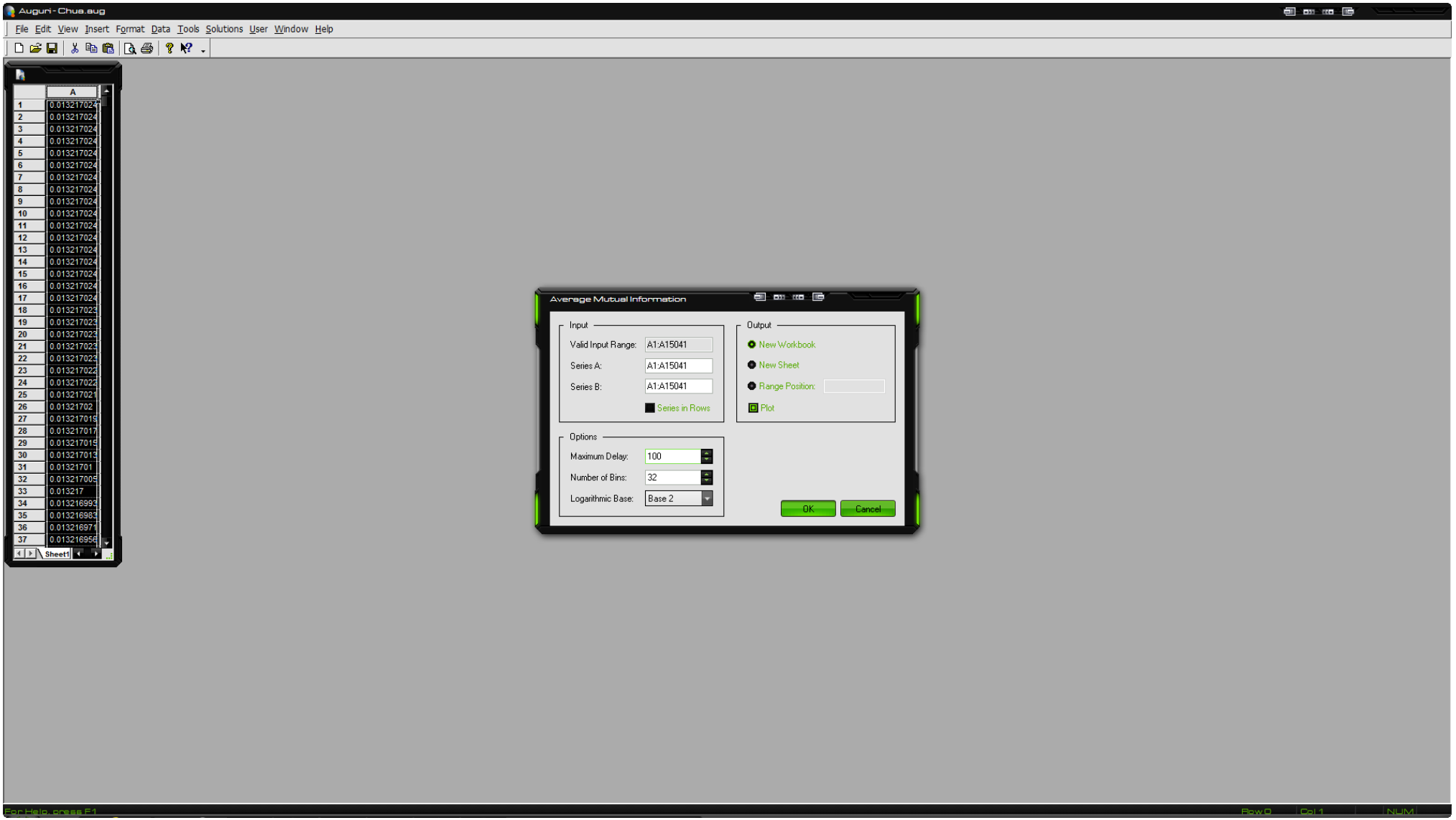


Fig. 18. Average Mutual Information Analysis Configuration.

Series A and B are set to the Valid Input Range's value, and Maximum Delay is set to "100".

Auguri - [Average Mutual Information.aug]

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1	Average Mutual Information																							
2																								
3	Logarithmic Base:	Base 2																						
4																								
5	Summary																							
6	Series A	Series B	First Minimum	Absolute Minimum																				
7	1	1	9	96																				
8																								
9	Results																							
10	Series A-1																							
11	Delay	Series B-1																						
12	0	4.6816140984594																						
13	1	3.1730035136951																						
14	2	2.4797014585213																						
15	3	2.023108200187																						
16	4	1.7069620440805																						
17	5	1.4846984199041																						
18	6	1.3256121573067																						
19	7	1.2136247011933																						
20	8	1.1463476164716																						
21	9	1.1154122941416																						
22	10	1.1191828259414																						
23	11	1.1520385645354																						
24	12	1.2202814514394																						
25	13	1.3300279340802																						
26	14	1.4923915930056																						
27	15	1.7488157964331																						
28	16	2.1451567283222																						
29	17	2.1184980698242																						
30	18	1.7694214436377																						
31	19	1.5233112022619																						
32	20	1.3483401640896																						
33	21	1.2058416056174																						
34	22	1.1021061362142																						
35	23	1.0266304980278																						
36	24	0.966995278278																						
37	25	0.93179975060564																						
38	26	0.92263449309109																						
39	27	0.93039769800007																						
40	28	0.93583945570127																						
41	29	0.9685955830846																						
42	30	1.0302761888775																						
43	31	1.1445832790149																						
44	32	1.3613948300382																						
45	33	1.548367075753																						
46	34	1.4195603707794																						
47	35	1.2726088848377																						
48	36	1.153867386534																						
49	37	1.0636012351001																						
50	38	0.9831053946459																						
51	39	0.91311249749376																						
52	40	0.84879936835001																						
53	41	0.80068178390379																						
54	42	0.77998195107643																						
55	43	0.76933405194381																						
56	44	0.7583918422191																						
57	45	0.7548792967643 3																						
58	46	0.7763925711316																						
59	47	0.83921561436994																						

Sheet1

For Help, press F1 Row 47 Col 1 NUM

Fig. 19. Average Mutual Information Results.

This window containing the Average Mutual Information results is then displayed; results are: First Minimum is “9” and Absolute Minimum is “96”.

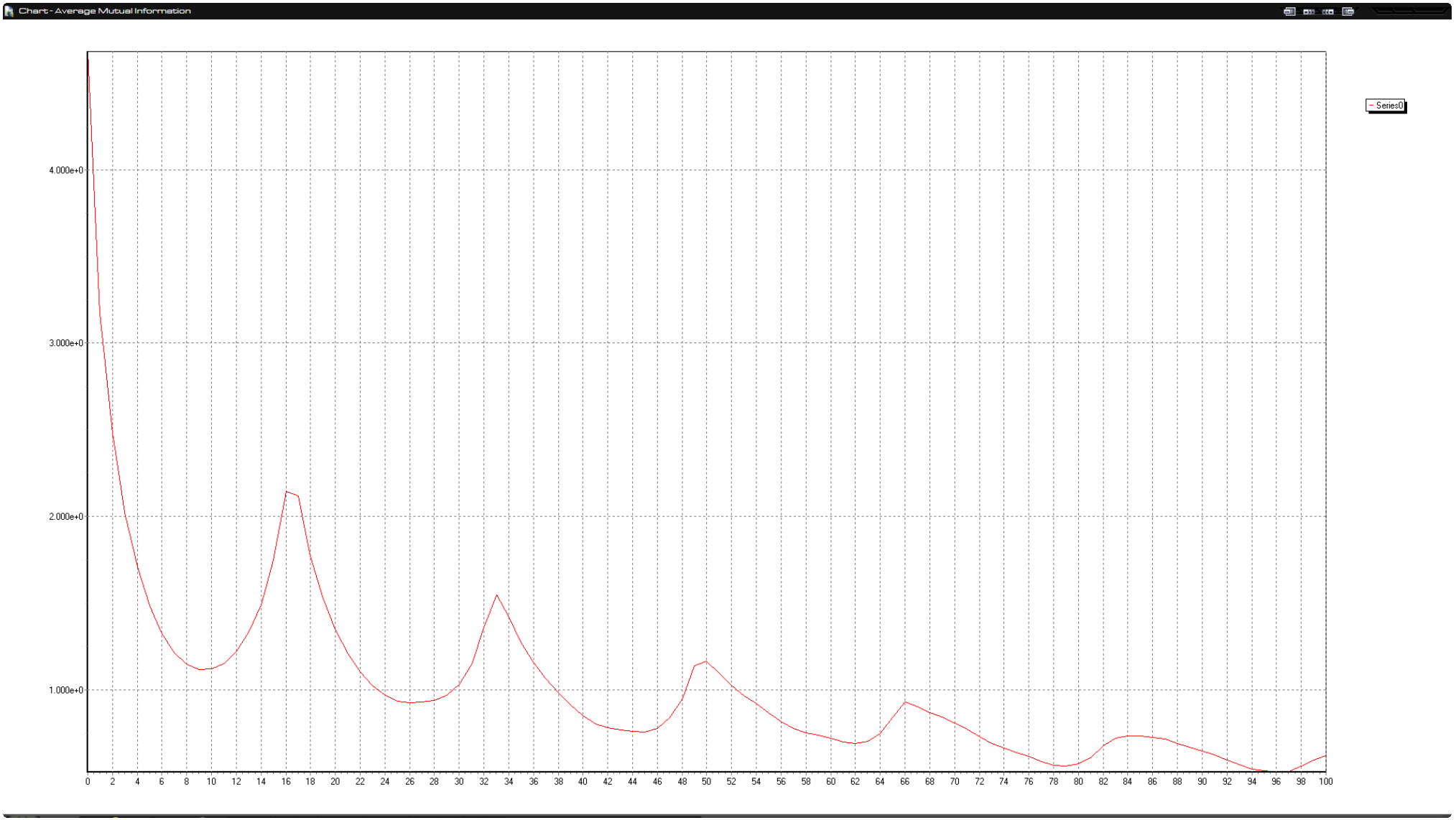


Fig. 20. Average Mutual Information Chart.

This graph confirms what was previously shown as raw data. It clearly shows that “9” is the first minimum as well as “96” is the Absolute one.

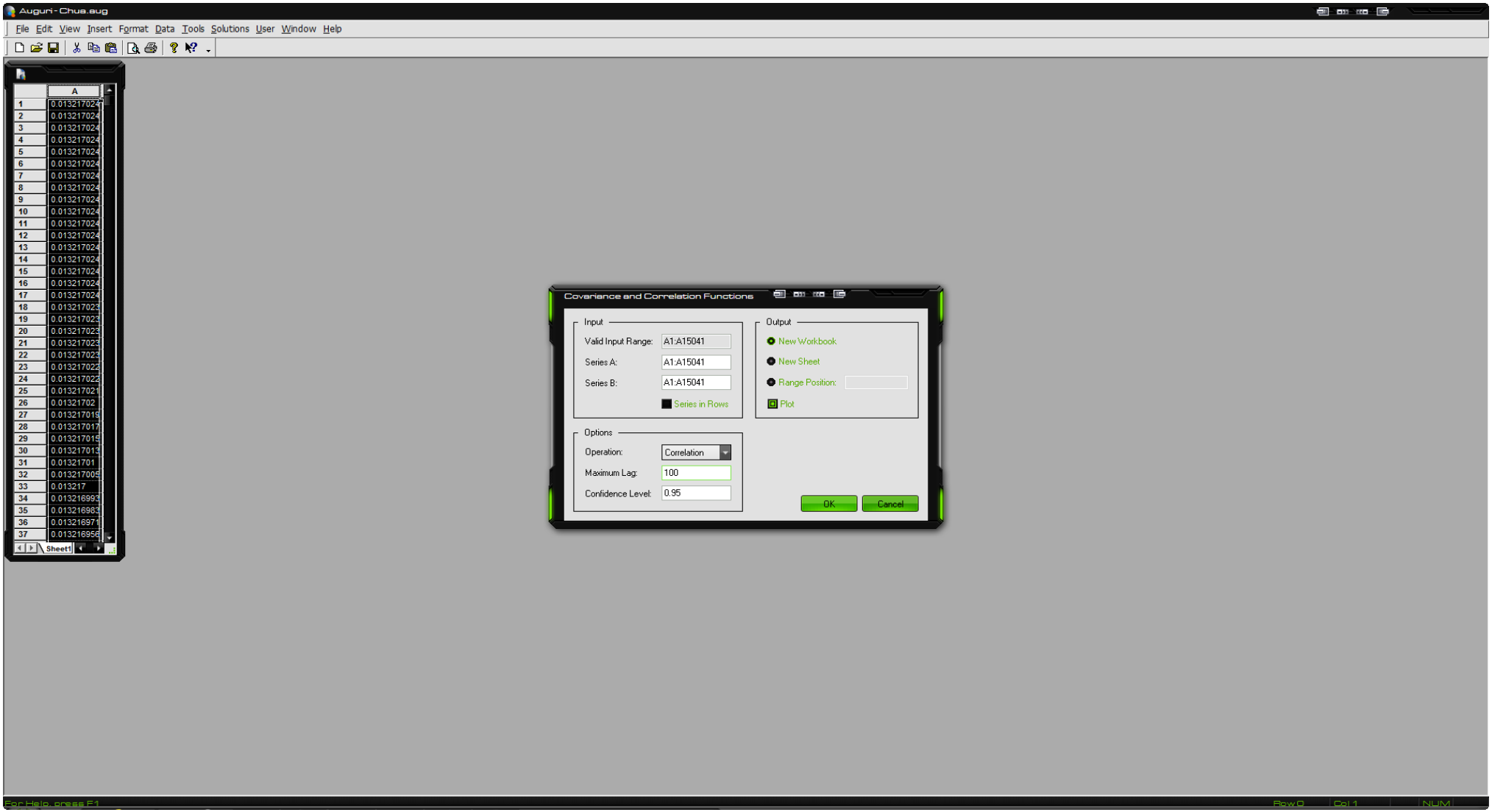
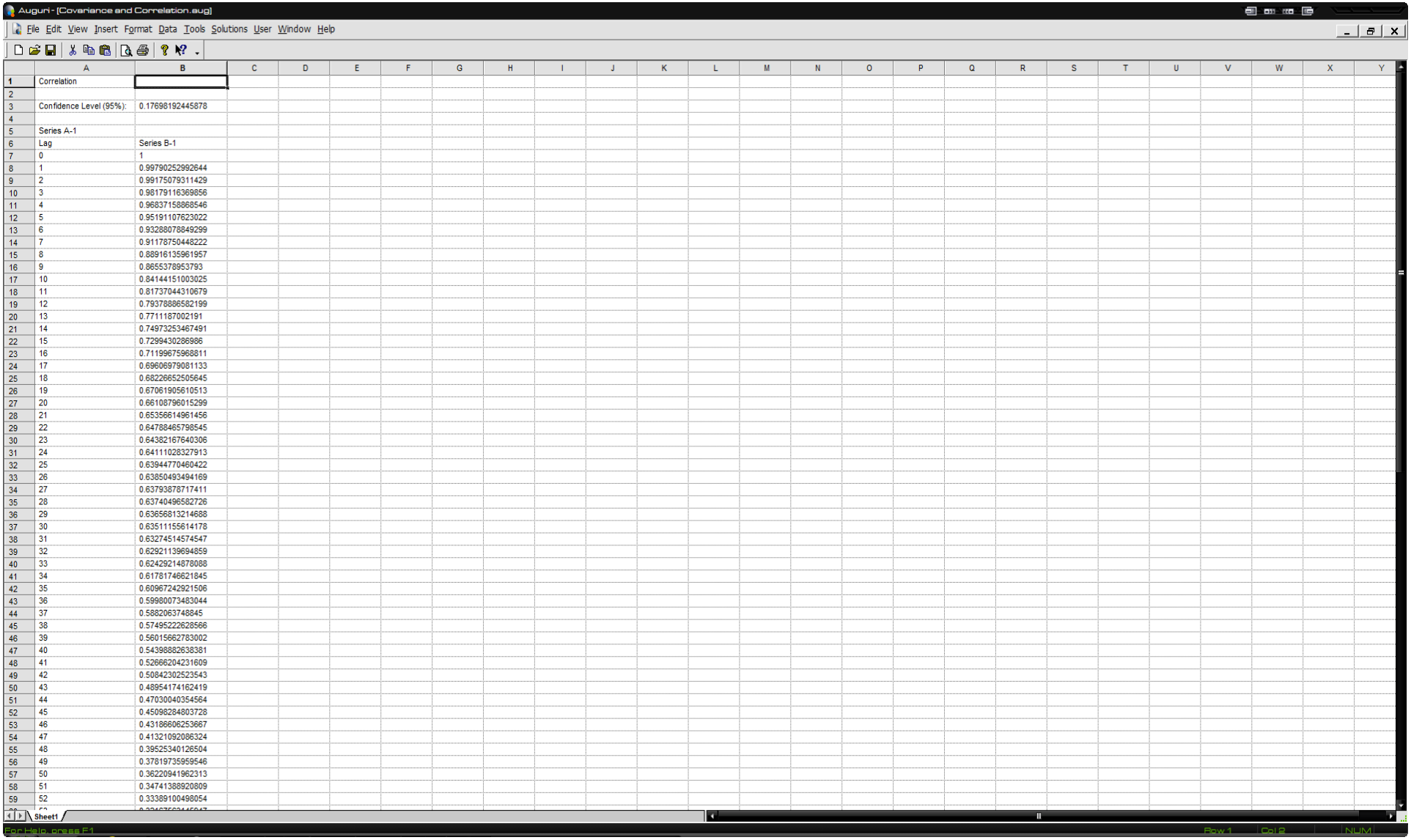


Fig. 21. Covariance and Correlation Configuration.

On the Covariance and Correlation, Series A and B are set to Valid Input Range's Value, and the Maximum Lag is set to "100".



	A	B
1	Correlation	
2		
3	Confidence Level (95%):	0.17698192445878
4		
5	Series A-1	
6	Lag	Series B-1
7	0	1
8	1	0.99790252992644
9	2	0.99175079311429
10	3	0.98179116369856
11	4	0.96837158868546
12	5	0.95191107623022
13	6	0.93288078849299
14	7	0.91178750448222
15	8	0.88916135961957
16	9	0.8655378953793
17	10	0.84144151003025
18	11	0.81737044310679
19	12	0.79378886582199
20	13	0.7711187002191
21	14	0.74973253467491
22	15	0.7299430286986
23	16	0.71199675968811
24	17	0.69606979081133
25	18	0.68226652505645
26	19	0.67061905610513
27	20	0.66108796015299
28	21	0.65356614961456
29	22	0.64788465798545
30	23	0.64382167640306
31	24	0.64111028327913
32	25	0.63944770460422
33	26	0.63850493494169
34	27	0.63793878717411
35	28	0.63740496582726
36	29	0.63656813214688
37	30	0.63511155614178
38	31	0.63274514574547
39	32	0.62921139694859
40	33	0.62429214878088
41	34	0.61781746621845
42	35	0.60967242921506
43	36	0.59980073483044
44	37	0.5882063748845
45	38	0.57495222628566
46	39	0.56015662783002
47	40	0.54398882638381
48	41	0.52666204231609
49	42	0.50842302523543
50	43	0.48954174162419
51	44	0.47030040354564
52	45	0.45098284803728
53	46	0.43186606253667
54	47	0.41321092086324
55	48	0.39525340126504
56	49	0.37819735959546
57	50	0.36220941962313
58	51	0.34741388920809
59	52	0.33389100498054

Fig. 22. Covariance and Correlation Results.

The Results of the analysis are then displayed on this window. Theiler Window's first zero is on "82".

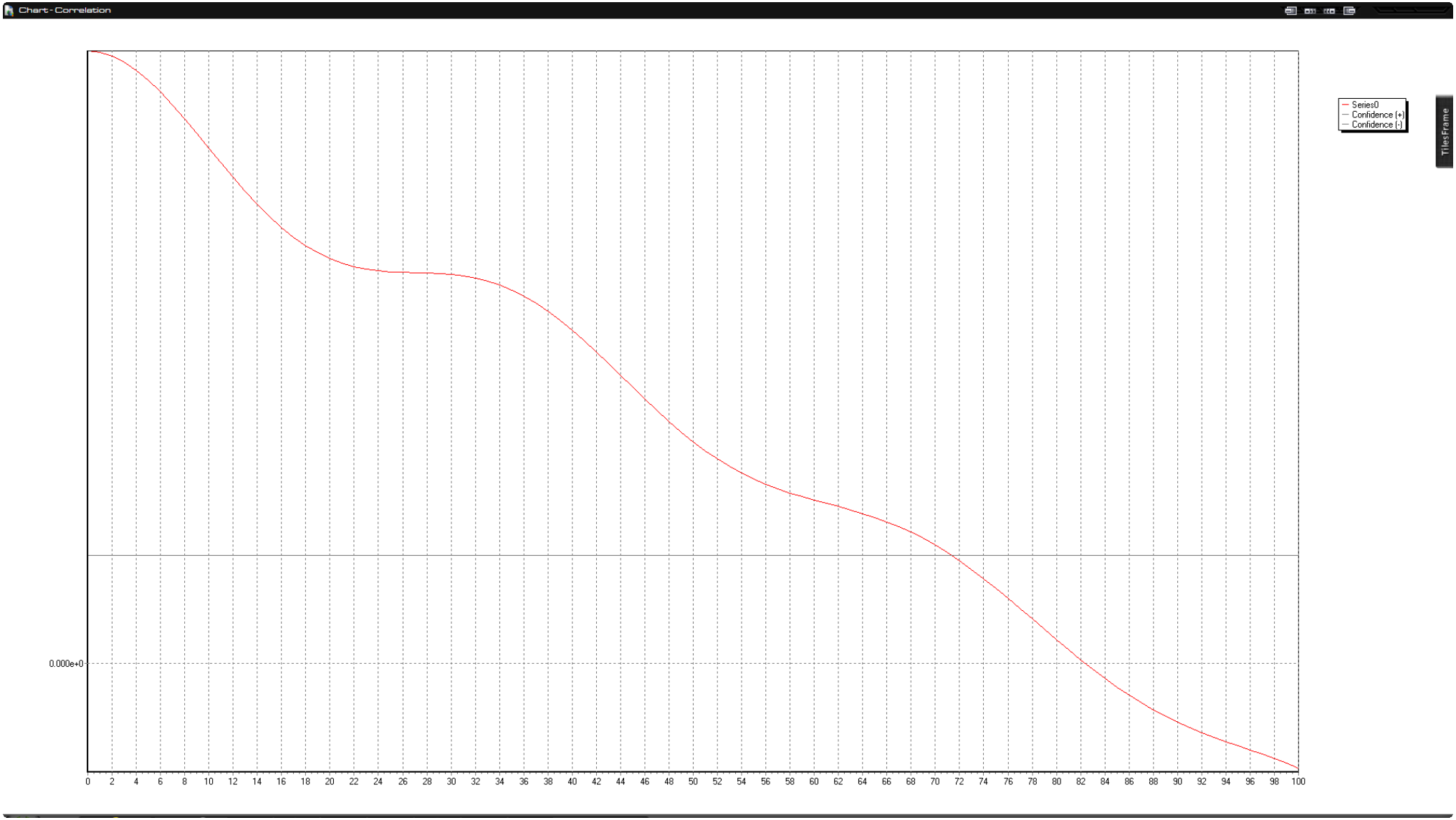


Fig. 23. Covariance and Correlation Chart.

As suggested by the aforementioned data, the first nullification on the graph (Theiler Window) is clearly to be found on “82”.

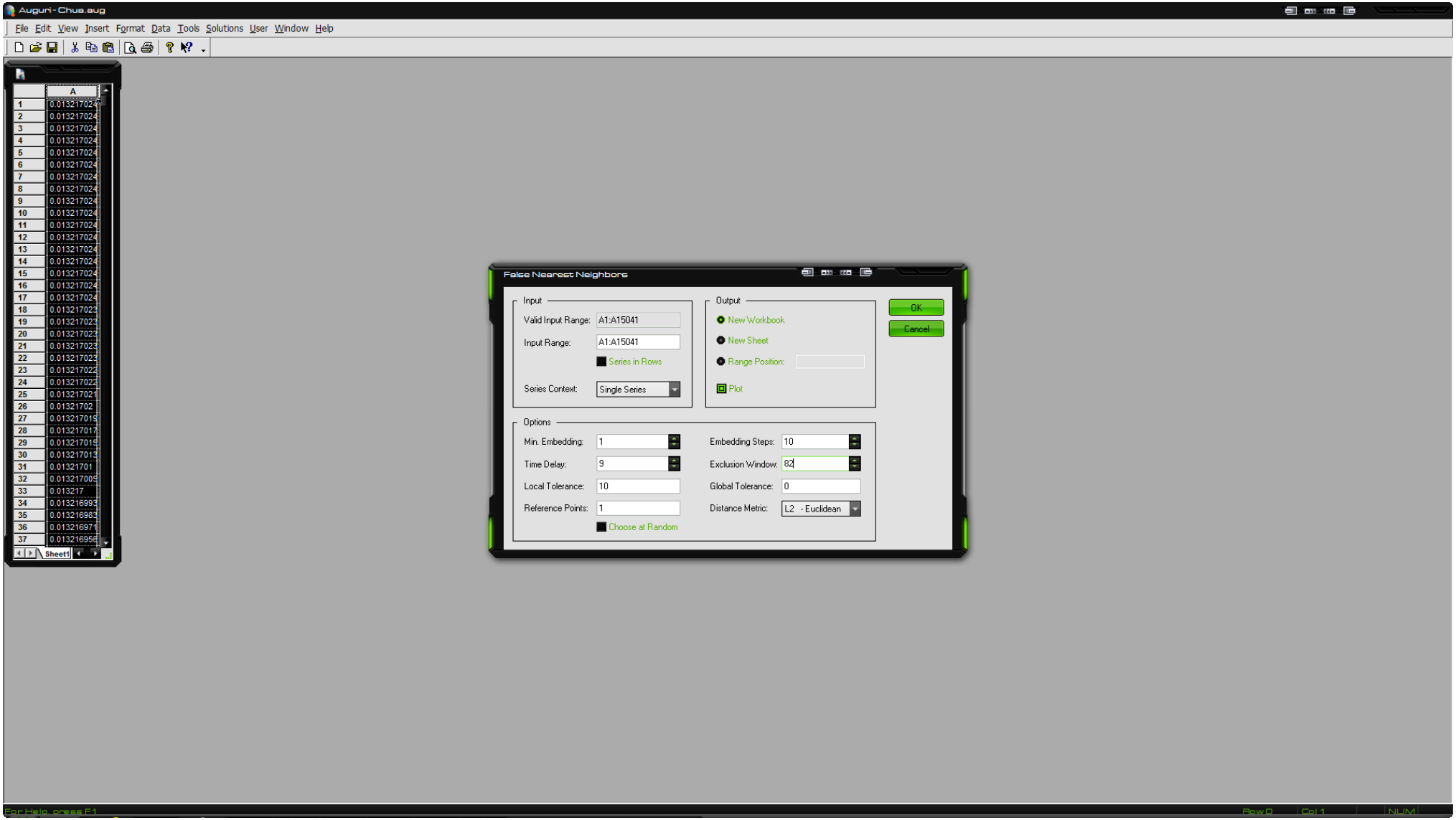
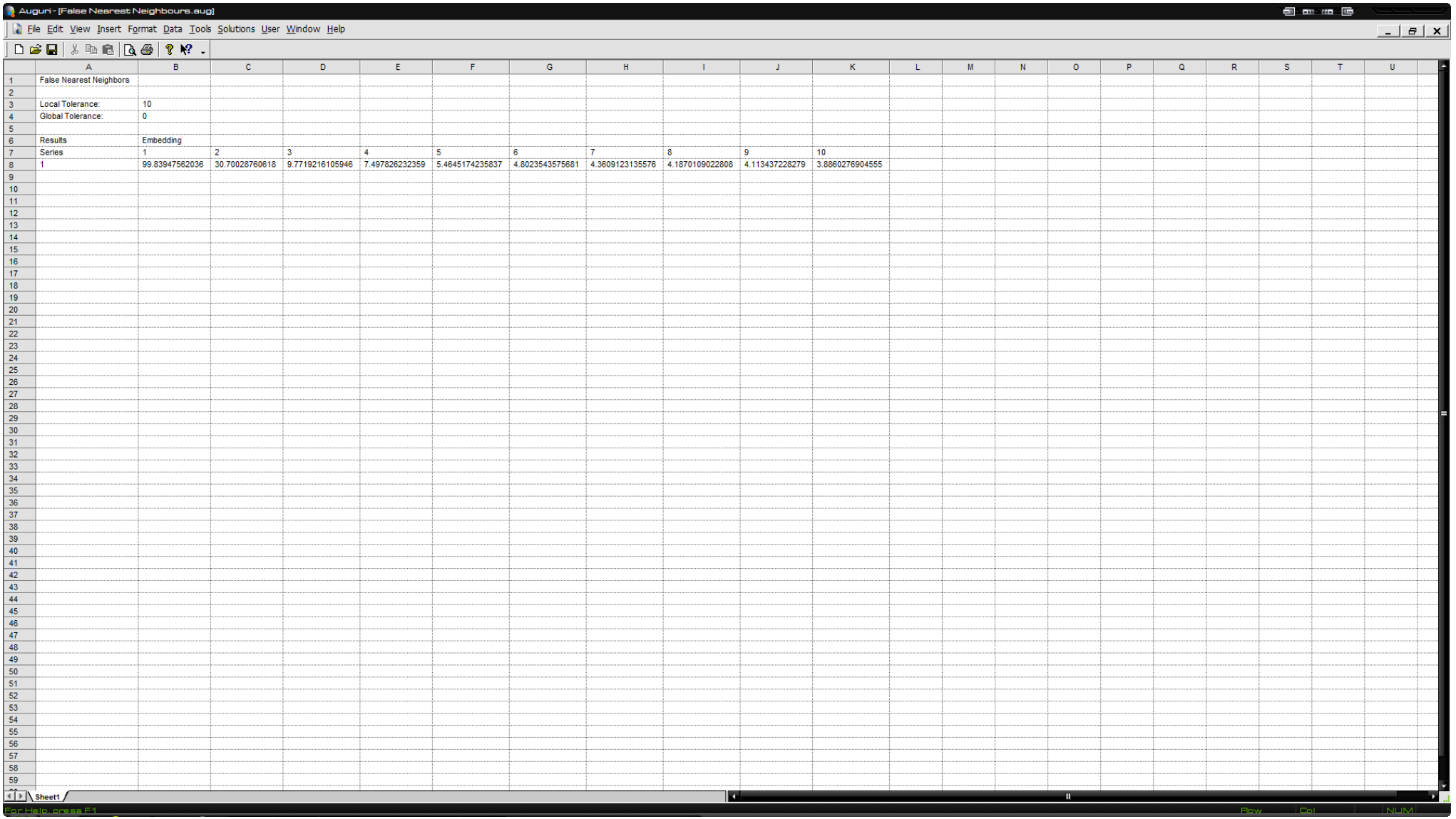


Fig. 24. False Nearest Neighbours Configuration.

For the False Nearest Neighbours Analysis the First Minimum of AMI is used as Time Delay, and First Zero of C&C as Exclusion Window.



	A	B	C	D	E	F	G	H	I	J	K
1	False Nearest Neighbors										
2											
3	Local Tolerance:	10									
4	Global Tolerance:	0									
5											
6	Results	Embedding									
7	Series	1	2	3	4	5	6	7	8	9	10
8	1	99.83947562036	30.70028760618	9.7719216105946	7.497826232359	5.4645174235837	4.8023543575681	4.3609123135576	4.1870109022808	4.113437228279	3.8860276904555

Fig. 25. False Nearest Neighbours Results.

After the Analysis is complete the results appear on this Window. Although there is no nullification, the value seems to be “4”.

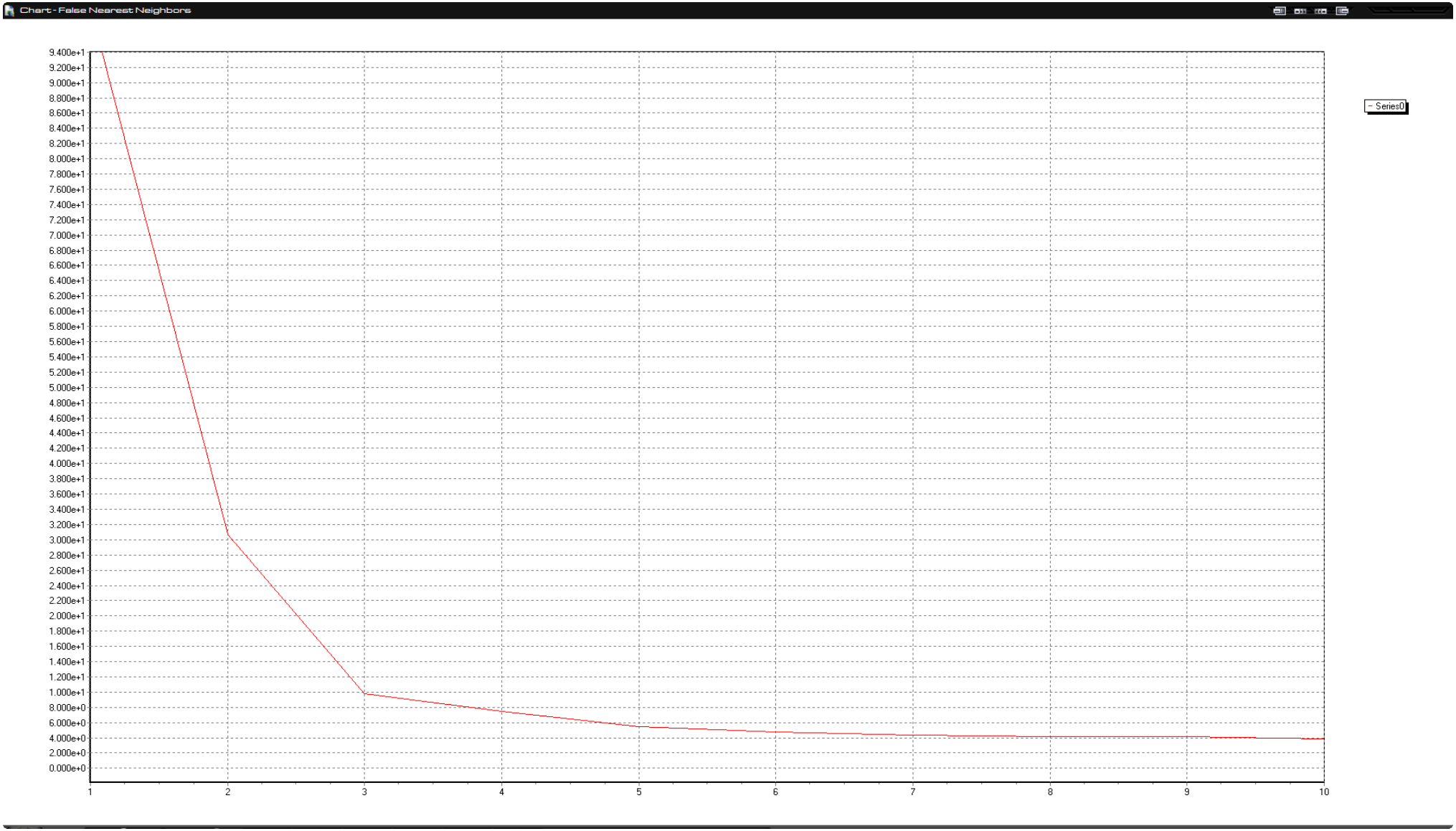


Fig. 26. False Nearest Neighbours Chart.

The False Nearest Neighbour's Graph corroborates with our previous findings of the value "4", therefore the system's dimension is four.

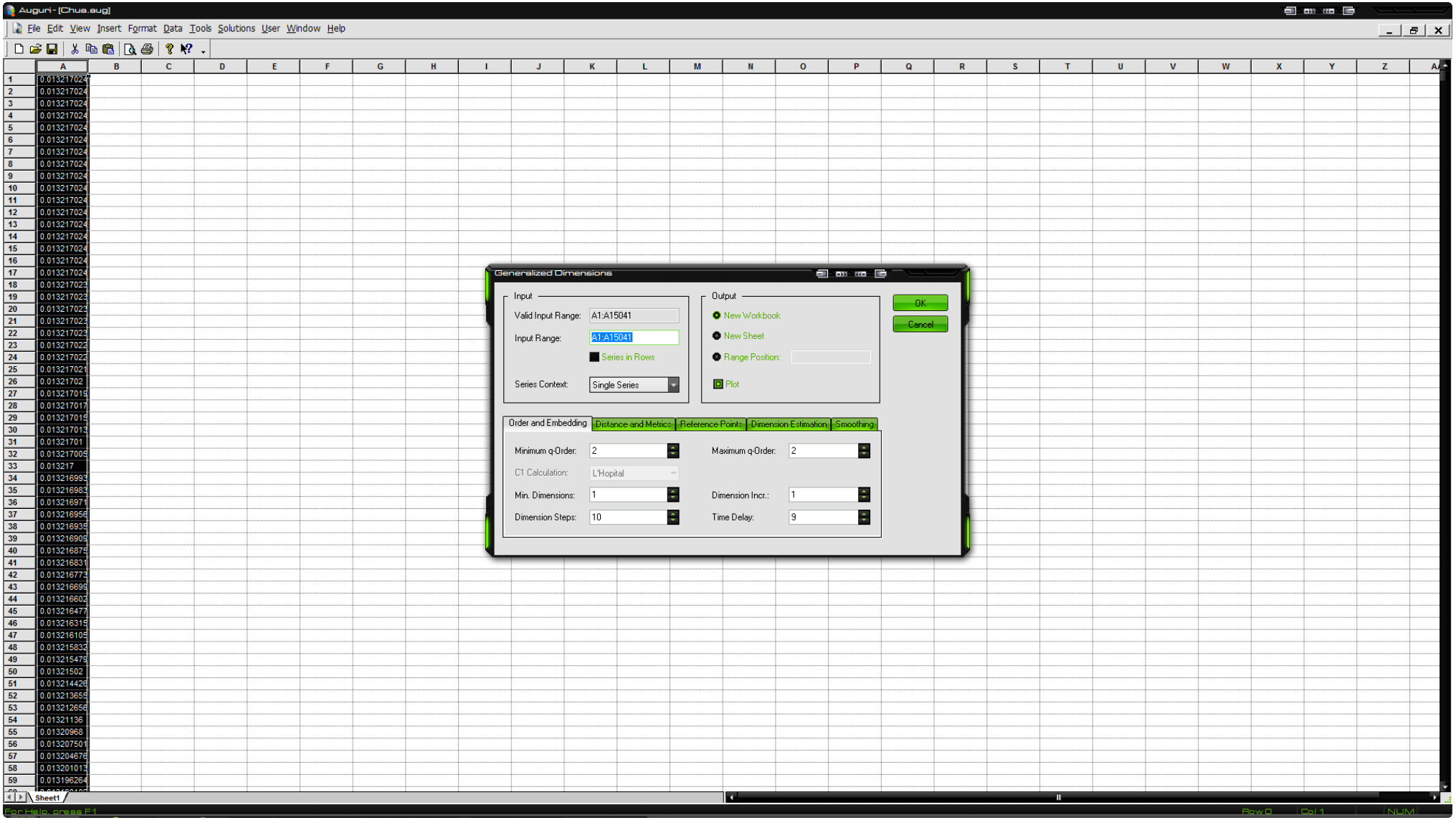


Fig. 27. Generalised Dimensions Configuration Tab 1.

Here, the Input Range is set to Valid Input Range's Value, the Dimension Steps to the Max Chaotic Value "10" and Time Delay to First Minimum.

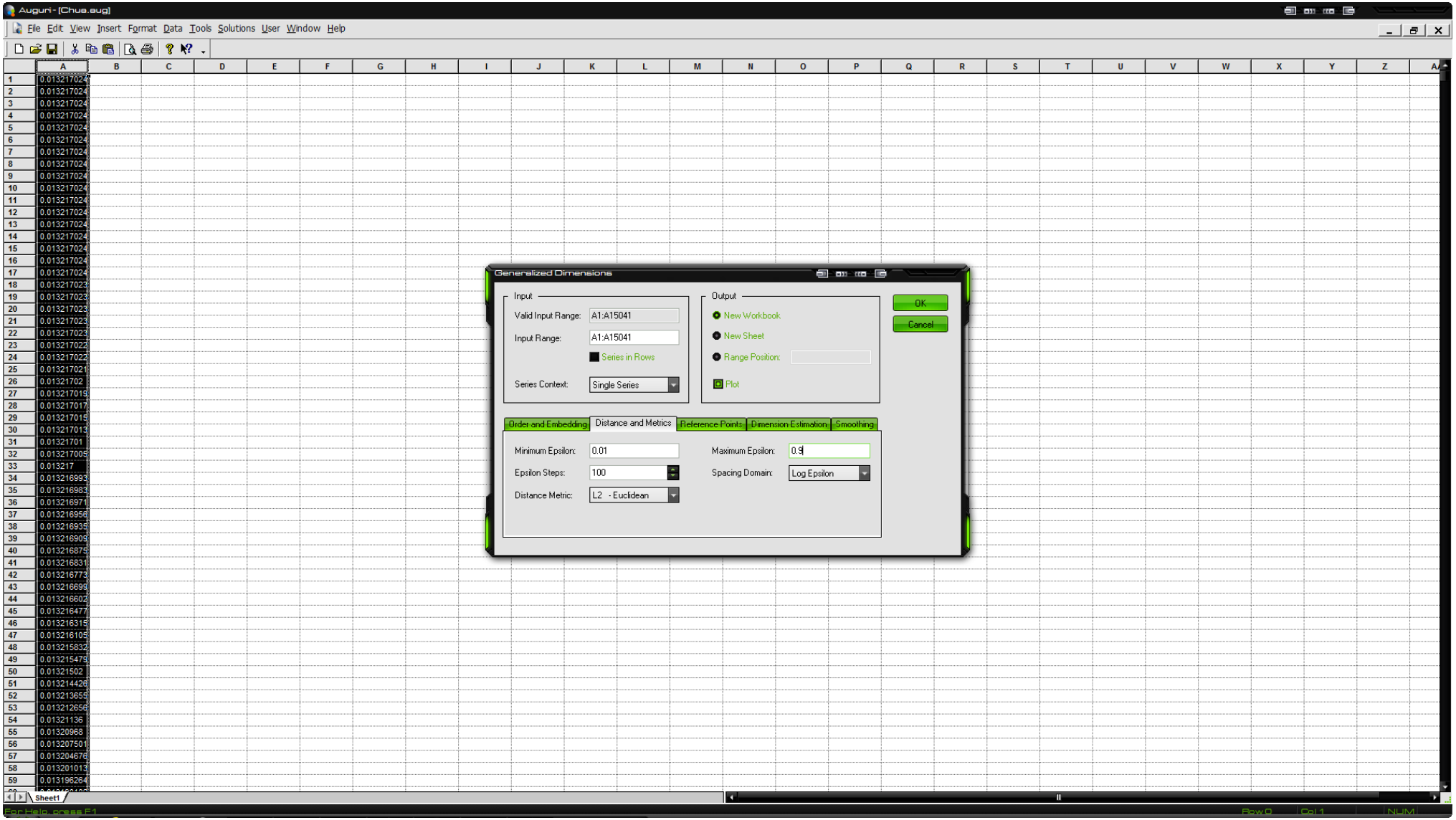


Fig. 28. Generalised Dimensions Configuration Tab 2.

On Distance and Metric, the Minimum Epsilon is set to 0.01 and the Maximum Epsilon to 0.9.

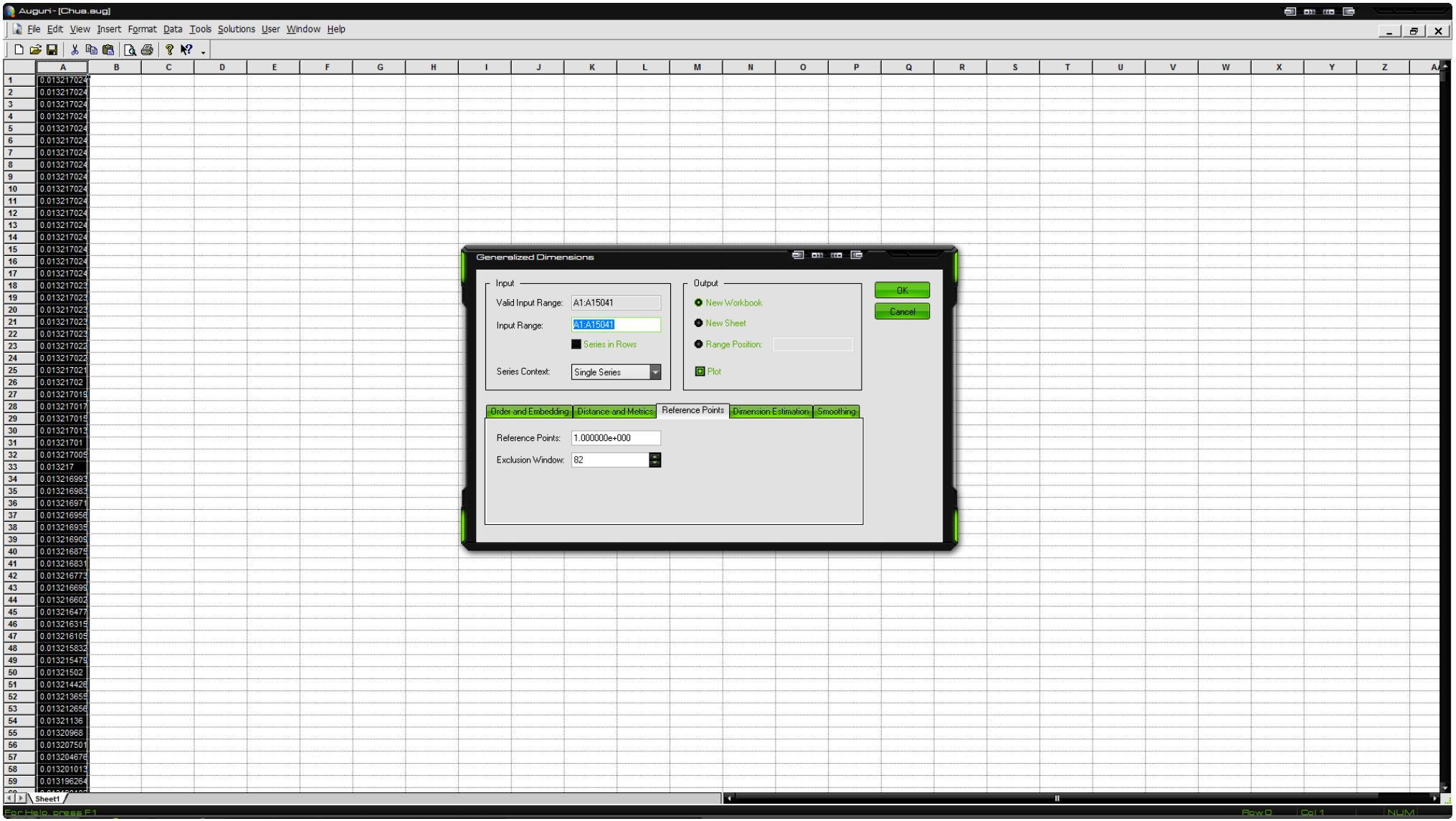
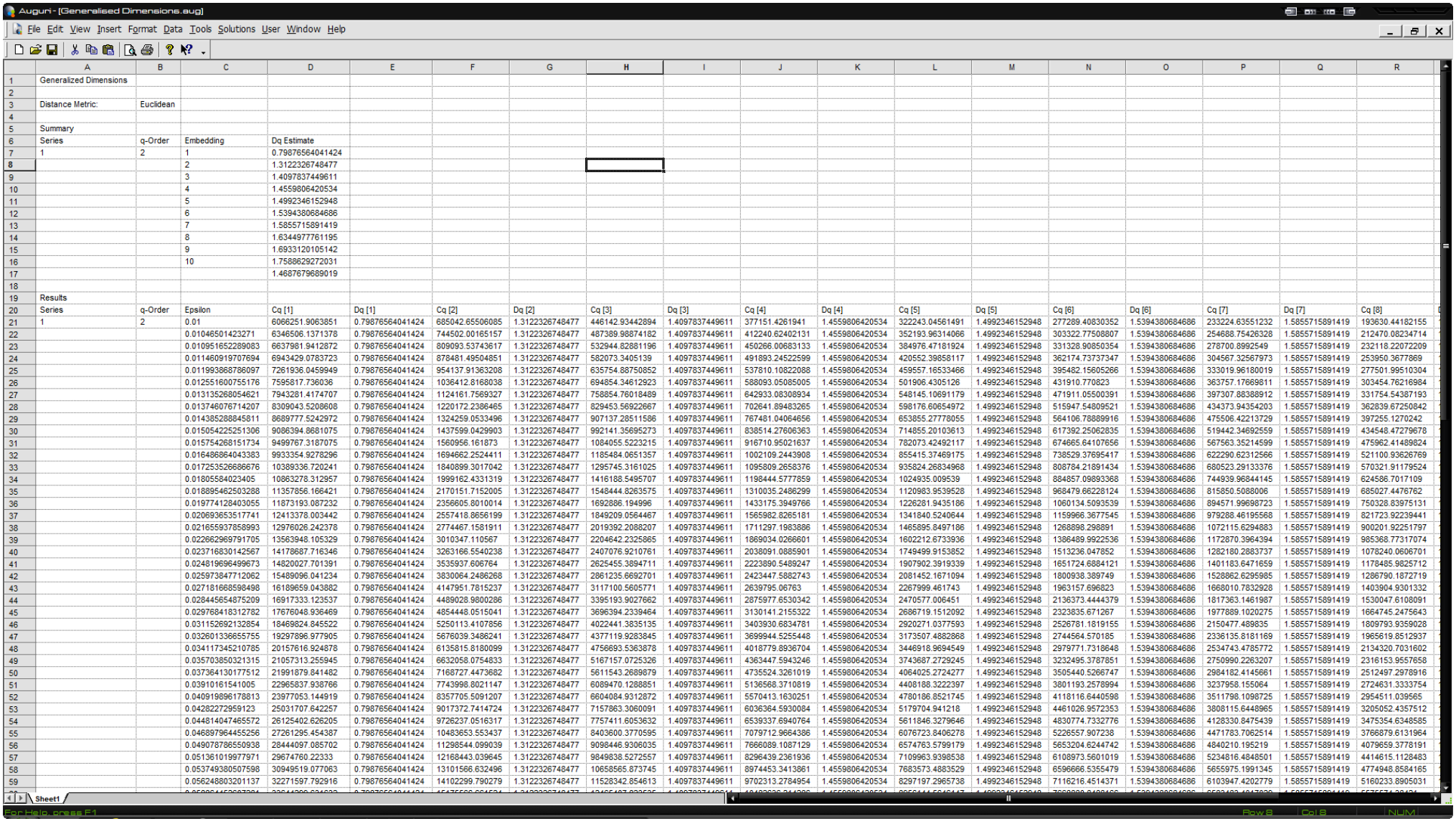


Fig. 29. Generalised Dimensions Configuration Tab 3.

On Reference Points, the Exclusion Window is set to the Theiler Window's Value and on Dimension Estimation and Smoothing no change takes place.



Auguri - [Generalised Dimensions.aug]

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Generalized Dimensions																	
2																		
3	Distance Metric:	Euclidean																
4																		
5	Summary																	
6	Series	q-Order	Embedding	Dq Estimate														
7	1	2	1	0.79876564041424														
8			2	1.3122326748477														
9			3	1.4097837449611														
10			4	1.4559806420534														
11			5	1.4992346152948														
12			6	1.5394380684686														
13			7	1.5855715891419														
14			8	1.6344977761195														
15			9	1.6933120105142														
16			10	1.7588629272031														
17				1.4687679689019														
18																		
19	Results																	
20	Series	q-Order	Epsilon	Cq [1]	Dq [1]	Cq [2]	Dq [2]	Cq [3]	Dq [3]	Cq [4]	Dq [4]	Cq [5]	Dq [5]	Cq [6]	Dq [6]	Cq [7]	Dq [7]	Cq [8]
21	1	2	0.01	6066251.9063851	0.79876564041424	685042.65506085	1.3122326748477	446142.93442894	1.4097837449611	377151.4261941	1.4559806420534	322243.04561491	1.4992346152948	277289.40830352	1.5394380684686	233224.63551232	1.5855715891419	193630.44182155
22			0.01046501423271	6346506.1371378	0.79876564041424	744502.00165157	1.3122326748477	487389.98874182	1.4097837449611	412240.62402131	1.4559806420534	352193.96314066	1.4992346152948	303322.77508807	1.5394380684686	254688.75426328	1.5855715891419	212470.08234714
23			0.010951652289083	6637981.9412872	0.79876564041424	809093.53743617	1.3122326748477	532944.82881196	1.4097837449611	450266.00683133	1.4559806420534	384976.47181924	1.4992346152948	331328.90850354	1.5394380684686	278700.8992549	1.5855715891419	232118.22072209
24			0.011460919707694	6943429.0783723	0.79876564041424	878481.49504851	1.3122326748477	582073.3405139	1.4097837449611	491893.24522599	1.4559806420534	420552.39858117	1.4992346152948	362174.73737347	1.5394380684686	304567.32567973	1.5855715891419	253950.3677869
25			0.011993868786097	7261936.0459949	0.79876564041424	954137.91363208	1.3122326748477	635754.88750852	1.4097837449611	537810.10822088	1.4559806420534	459557.16533466	1.4992346152948	395482.15605266	1.5394380684686	333019.96180019	1.5855715891419	277501.99510304
26			0.012551600755176	7595817.736036	0.79876564041424	1036412.8168038	1.3122326748477	694854.34612923	1.4097837449611	588093.05085005	1.4559806420534	501906.4305126	1.4992346152948	431910.770823	1.5394380684686	363757.17669811	1.5855715891419	303454.76216984
27			0.013135268054621	7943281.4174707	0.79876564041424	1124161.7569327	1.3122326748477	758854.76018489	1.4097837449611	642933.08308934	1.4559806420534	548145.10691179	1.4992346152948	471911.05500391	1.5394380684686	397307.88388912	1.5855715891419	331754.54387193
28			0.013746076714207	8309043.5208608	0.79876564041424	1220172.2386465	1.3122326748477	829453.56922667	1.4097837449611	702641.89483265	1.4559806420534	598176.60654972	1.4992346152948	515947.54809521	1.5394380684686	434373.94354203	1.5855715891419	362839.67250842
29			0.014385288845811	8689777.5242972	0.79876564041424	1324259.0533496	1.3122326748477	907137.28511586	1.4097837449611	767481.04064656	1.4559806420534	653855.27778055	1.4992346152948	564106.78889916	1.5394380684686	475506.42213729	1.5855715891419	397255.1270242
30			0.015054225251306	9086394.8681075	0.79876564041424	1437599.0429903	1.3122326748477	992141.35695273	1.4097837449611	838514.27606363	1.4559806420534	714855.20103613	1.4992346152948	617392.25062835	1.5394380684686	519442.34692559	1.5855715891419	434548.47279678
31			0.015754268151734	9499767.3187075	0.79876564041424	1560956.161873	1.3122326748477	1084055.5223215	1.4097837449611	916710.95021637	1.4559806420534	782073.42492117	1.4992346152948	674665.64107656	1.5394380684686	567563.35214599	1.5855715891419	475962.41489824
32			0.016486864043383	9933354.9278296	0.79876564041424	1694662.2524411	1.3122326748477	1185484.0651357	1.4097837449611	1002109.2443908	1.4559806420534	855415.37469175	1.4992346152948	738529.37695417	1.5394380684686	622290.62312566	1.5855715891419	521100.93626769
33			0.017253526686676	10389336.720241	0.79876564041424	1840899.3017042	1.3122326748477	1295745.3161025	1.4097837449611	1095809.2658376	1.4559806420534	935824.26834968	1.4992346152948	808784.21891434	1.5394380684686	680523.29133376	1.5855715891419	570321.91179524
34			0.01805584023405	10863278.312957	0.79876564041424	1999162.4331319	1.3122326748477	1416188.5495707	1.4097837449611	1198444.5777859	1.4559806420534	1024935.009539	1.4992346152948	884857.09893368	1.5394380684686	744939.96844145	1.5855715891419	624586.7017109
35			0.018895462503288	11357856.166421	0.79876564041424	2170151.7152005	1.3122326748477	1548444.8263575	1.4097837449611	1310035.2486299	1.4559806420534	1120983.9539528	1.4992346152948	968479.66228124	1.5394380684686	815850.5088006	1.5855715891419	685027.4476762
36			0.019774128403055	11873193.087232	0.79876564041424	2356605.8010014	1.3122326748477	1692886.194996	1.4097837449611	1433175.3949766	1.4559806420534	1226281.9435186	1.4992346152948	1060134.5093539	1.5394380684686	894571.99698723	1.5855715891419	750328.83975131
37			0.020693653517741	12413378.003442	0.79876564041424	2557418.8656199	1.3122326748477	1849209.0564467	1.4097837449611	1565982.8265181	1.4559806420534	1341840.5240644	1.4992346152948	1159966.3677545	1.5394380684686	979288.46195568	1.5855715891419	821723.92239441
38			0.021655937858993	12976026.242378	0.79876564041424	2774467.1581911	1.3122326748477	2019392.2088207	1.4097837449611	1711297.1983886	1.4559806420534	1465895.8497186	1.4992346152948	1268898.298891	1.5394380684686	1072115.6294883	1.5855715891419	900201.92251797
39			0.022662969791705	13563948.105329	0.79876564041424	3010347.110567	1.3122326748477	2204642.2325865	1.4097837449611	1869034.0266601	1.4559806420534	1602212.6733936	1.4992346152948	1386489.9922536	1.5394380684686	1172870.3964394	1.5855715891419	985368.77317074
40			0.023716830142567	14178687.716346	0.79876564041424	3263166.5540238	1.3122326748477	2407076.9210761	1.4097837449611	2038091.0885901	1.4559806420534	1749499.9153852	1.4992346152948	1513236.047852	1.5394380684686	1282180.2883737	1.5855715891419	1078240.0606701
41			0.024819696499673	14820027.701391	0.79876564041424	3535937.606764	1.3122326748477	2625455.3894711	1.4097837449611	2223890.5489247	1.4559806420534	1907902.3919339	1.4992346152948	1651724.6884121	1.5394380684686	1401183.6471659	1.5855715891419	1178485.9825712
42			0.025973847712062	15489096.041234	0.79876564041424	3830064.2486268	1.3122326748477	2861235.6692701	1.4097837449611	2423447.5882743	1.4559806420534	2081452.1671094	1.4992346152948	1800938.389749	1.5394380684686	1528862.6295985	1.5855715891419	1286790.1872719
43			0.027181668598498	16189659.043882	0.79876564041424	4147951.7815237	1.3122326748477	3117100.5605771	1.4097837449611	2639795.06763	1.4559806420534	2267999.461743	1.4992346152948	1963157.696823	1.5394380684686	1668010.7832928	1.5855715891419	1403904.9301332
44			0.028445654875209	16917333.123537	0.79876564041424	4489028.9800286	1.3122326748477	3395193.9027662	1.4097837449611	2875977.6530342	1.4559806420534	2470577.006451	1.4992346152948	2136373.4444379	1.5394380684686	1817363.1461987	1.5855715891419	1530047.6108091
45			0.029768413212782	17676048.936469	0.79876564041424	4854448.0515041	1.3122326748477	3696394.2339464	1.4097837449611	3130141.2155322	1.4559806420534	2686719.1512092	1.4992346152948	2323835.671267	1.5394380684686	1977889.1020275	1.5855715891419	1664745.2475643
46			0.031152692132854	18469824.845522	0.79876564041424	5250113.4107856	1.3122326748477	4022441.3835135	1.4097837449611	3403930.6834781	1.4559806420534	2920271.0377593	1.4992346152948	2526781.1819155	1.5394380684686	2150477.489835	1.5855715891419	1809793.9359028
47			0.032601336655755	19297896.977905	0.79876564041424	5676039.3486241	1.3122326748477	4377119.9283845	1.4097837449611	3699944.5255448	1.4559806420534	3173507.4882868	1.4992346152948	2744564.570185	1.5394380684686	2336135.8181169	1.5855715891419	1965619.8512937
48			0.034117345210785	20157616.924878	0.79876564041424	6135815.8180099	1.3122326748477	4756693.5363878	1.4097837449611	4018779.8936704	1.4559806420534	3446918.9694549	1.4992346152948	2979771.7318648	1.5394380684686	2534743.4785772	1.5855715891419	2134320.7031602
49			0.035703850321315	21057313.255945	0.79876564041424	6632058.0754833	1.3122326748477	5167157.0725326	1.4097837449611	4363447.5943246	1.4559806420534	3743687.2729245	1.4992346152948	3232495.3787851	1.5394380684686	2750990.2263207	1.5855715891419	2316153.9557658
50			0.037364130177512	21991879.841482	0.79876564041424	7168727.4473682	1.3122326748477	5611543.2689879	1.4097837449611	4735524.3261019	1.4559806420534	4064025.2724277	1.4992346152948	3505440.5266747	1.5394380684686	2984182.4145661	1.5855715891419	2512497.2978916
51			0.03910161541005	22965837.938766	0.79876564041424	7743998.8021147	1.3122326748477	6089470.1288851	1.4097837449611	5136568.3710819	1.4559806420534	4408188.3222397	1.4992346152948	3801193.2578994	1.5394380684686	3237958.155064	1.5855715891419	2724631.3333754
52			0.040919896178813	23977053.144919	0.79876564041424	8357705.5091207	1.3122326748477	6604084.9312872	1.4097837449611	5570413.1630251	1.4559806420534	4780186.8521745	1.4992346152948	4118116.6440598	1.5394380684686	3511798.1098725	1.5855715891419	2954511.039565
53			0.04282272959123	25031707.642257	0.79876564041424	9017372.7414724	1.3122326748477	7157863.3060091	1.4097837449611	6036364.5930084	1.4559806420534	5179704.941218	1.4992346152948	4461026.9572353	1.5394380684686	3808115.6448965	1.5855715891419	3205052.4357512
54			0.044814047465572	26125402.626205	0.79876564041424	9726237.0516317	1.3122326748477	7757411.6053632	1.4097837449611	6539337.6940764	1.4559806420534	5611846.3279646	1.4992346152948	4830774.7332776	1.5394380684686	4128330.8475439	1.5855715891419	3475354.6348585
55			0.046897964455256	27261295.454387	0.79876564041424	10483653.553437	1.3122326748477	8403600.3770595	1.4097837449611	7079712.9664386	1.4559806420534	6076723.8406278	1.4992346152948	5226557.907238	1.5394380684686	4471783.7062514	1.5855715891419	3766879.6131964
56			0.049078786550938	28444097.085702	0.79876564041424	11298544.099039	1.3122326748477	9098446.9306035	1.4097837449611	7666089.1087129	1.4559806420534	6574763.5799179	1.4992346152948	5653204.6244742	1.5394380684686	4840210.195219	1.5855715891419	4079659.3778191
57			0.051361019977971	29674760.22333	0.79876564041424	12168443.039645	1.3122326748477	9849838.5272557	1.4097837449611	8296439.2361936	1.4559806420534	7109963.9398538	1.4992346152948	6108973.5601019	1.5394380684686	5234816.4848501	1.5855715891419	4414615.1128483
58			0.053749380507598	30949519.077063	0.79876564041424	13101566.632496	1.3122326748477	10658565.873745	1.4097837449611	8974453.3413861	1.4559806420534	7683573.4883529	1.4992346152948	6596666.5355479	1.5394380684686	5655975.1991345	1.5855715891419	4774948.8584165
59			0.056248803201137	32271597.792916	0.79876564041424	14102299.790279	1.3122326748477	11528342.854613	1.4097837449611	9702313.2784954	1.4559806420534	8297197.2965738	1.4992346152948	7116216.4514371	1.5394380684686	6103947.4202779	1.5855715891419	5160233.8905031

Sheet1

For Help, press F1 Row 8 Col 8 NUM

Fig. 30. Generalised Dimensions Results

The results from Generalised Dimensions appear on this Window. Generalised Dimensions are followed by three Graphs.

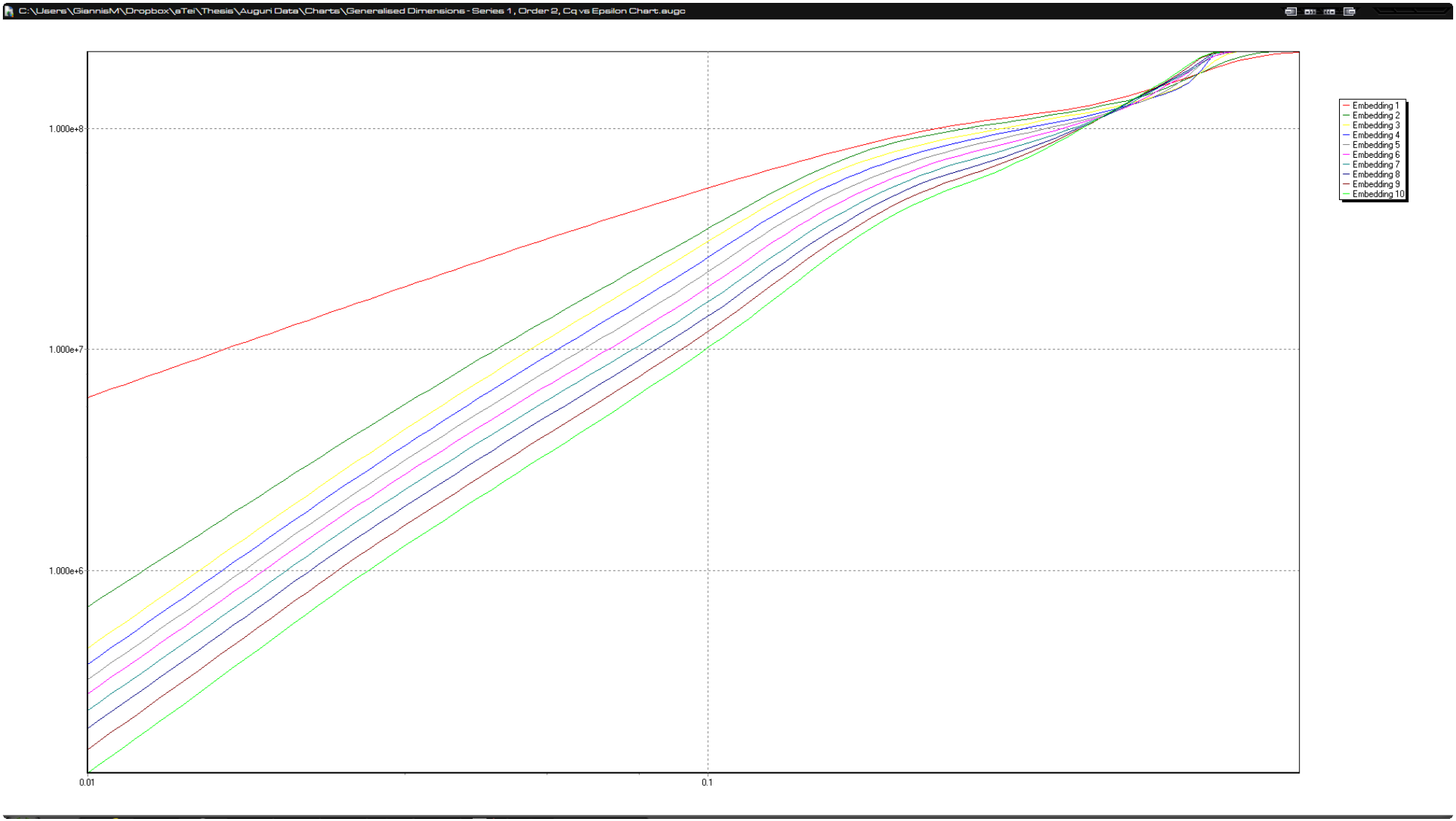

Fig. 31. Generalised Dimensions - Series 1, Order 2: Cq vs. Epsilon Chart.

The Least Square Method is used on the slopes of these lines to produce the Series 1, Order 2: Dq vs. Embedding Dimension diagram [Fig. 33.].

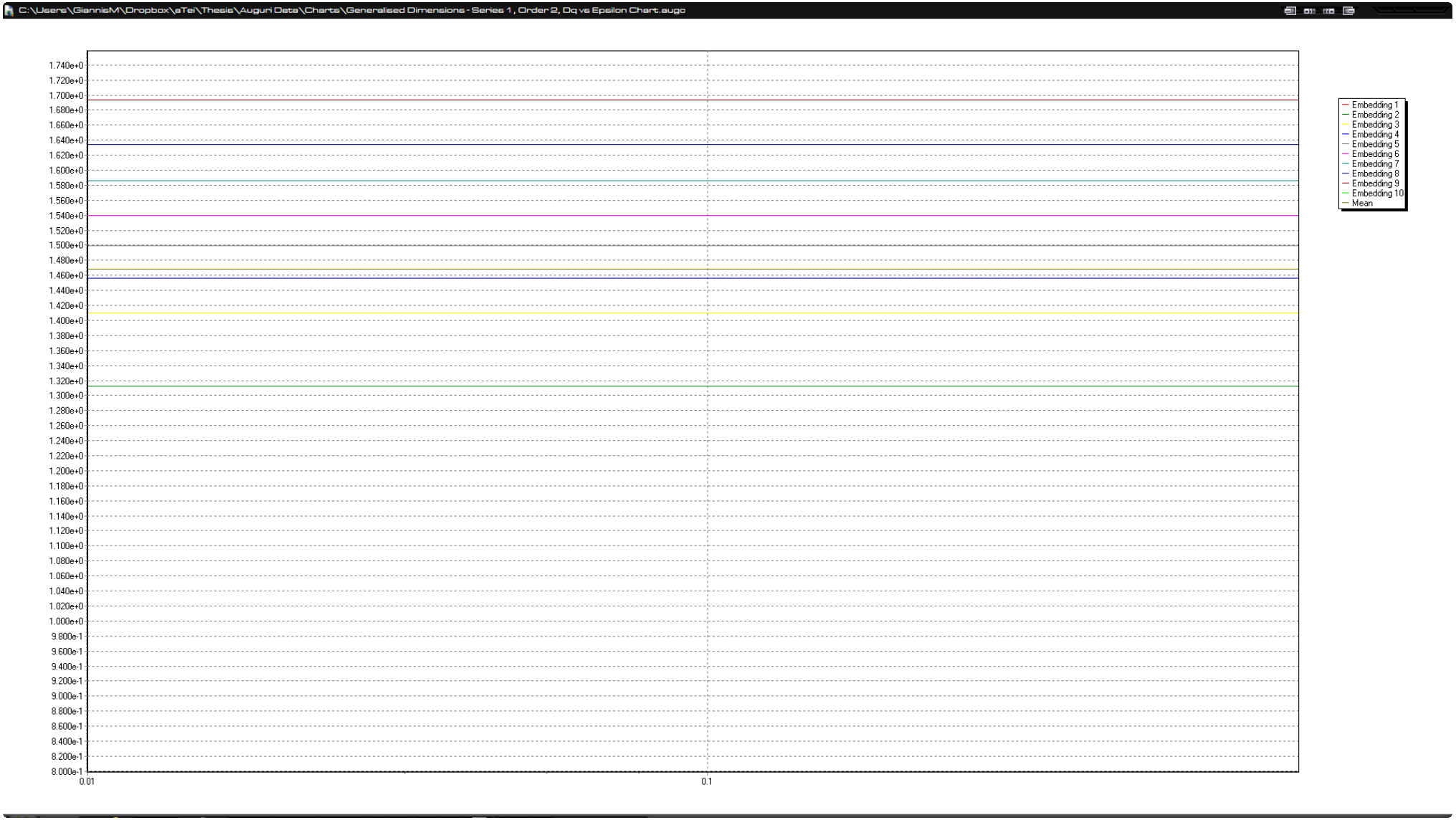

Fig. 32. Generalised Dimensions - Series 1, Order 2: Dq vs. Epsilon Chart.
This chart depicts the derivatives (slopes) of [Fig. 31.].

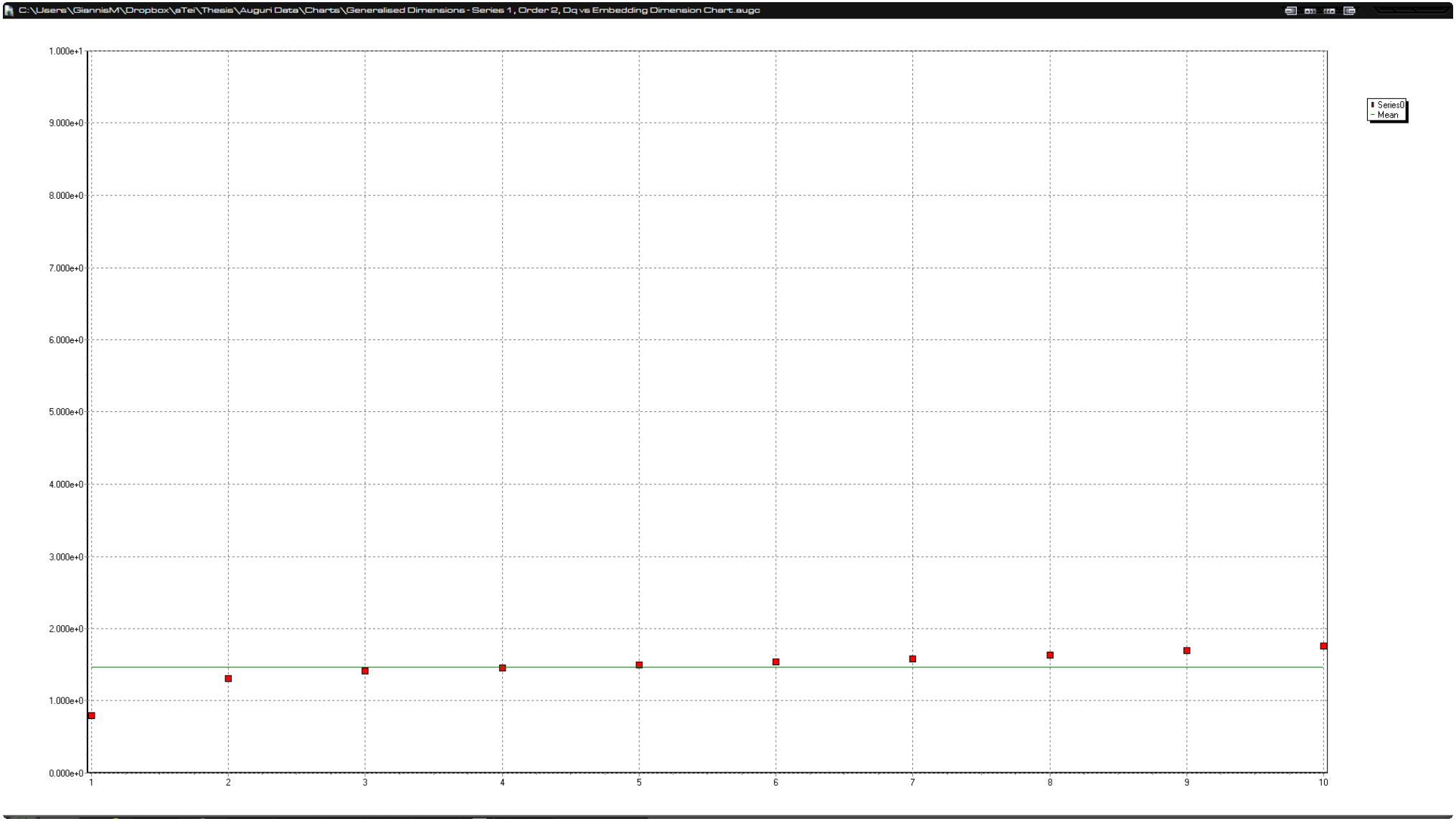

Fig. 33. Generalised Dimensions - Series 1, Order 2: Dq vs. Embedding Dimension Chart.

This chart shows that the Fractal Dimension of the system is FD ≈ 1.8, which is indicative of chaos.

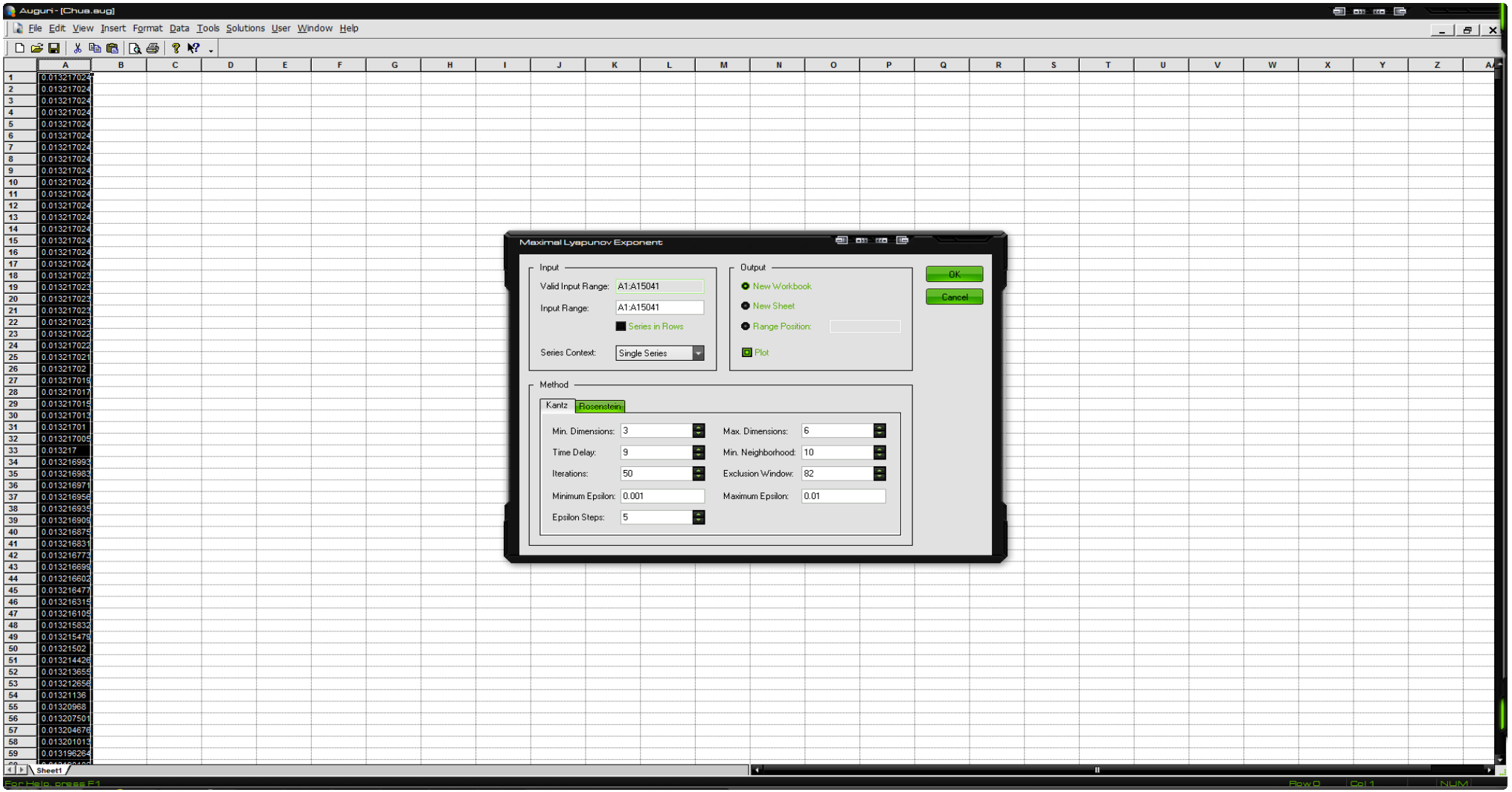

Fig. 34. Maximum Lyapunov Exponent Configuration Tab 1.

On the Maximum Lyapunov Exponent the “Min Dimensions” is set to “3”, the “Max Dimensions” is set to “6”, the Time Delay to the First minimum, Minimum Neighbours to 10 and the Exclusion Window to the Theiler Window’s Value.

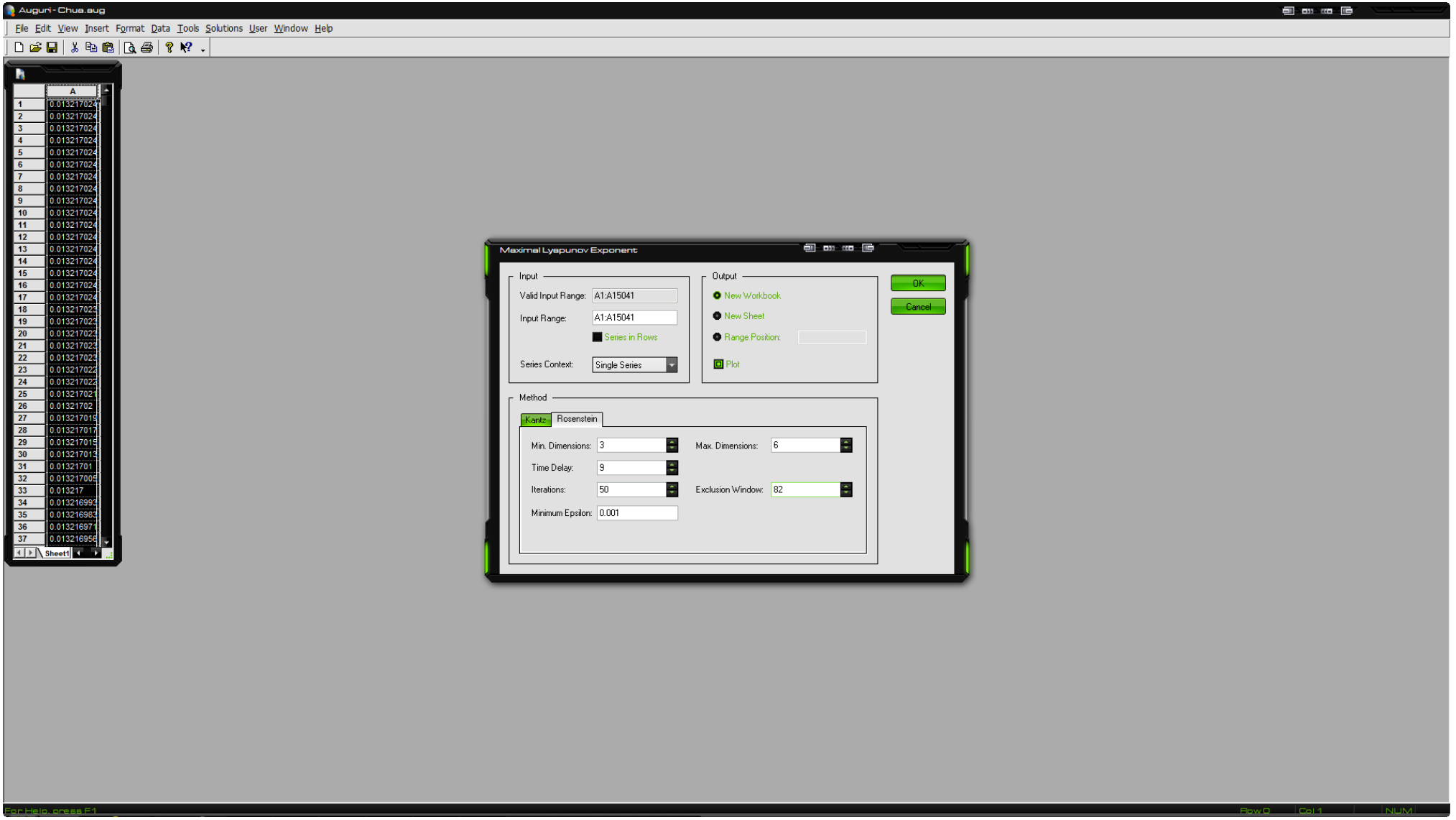

Fig. 35. Maximum Lyapunov Exponent Configuration Tab 2.

On the Rosenstein Tab the “Min Dimensions” is set to “3”, “Max Dimensions” is set to “6”, Time Delay to “9”, and the Exclusion Window to “82”.

Auguri - [Maximum Lyapunov Exponent.aug]

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	A	B	C	D	E	F	G	H	I
1	Maximal Lyapunov Exponent								
2									
3	Method:	Rosenstein							
4									
5	Series	Dimensions	Epsilon Step	Iteration	Divergence Log	Point Pairs		Series	Estimated Lyapunov
6	1	3	1	0	-5.5473258761697	14946		1	0.032414512233909
7				1	-5.4306238550797	14945			
8				2	-5.3136914526271	14944			
9				3	-5.223205043828	14944			
10				4	-5.1547622791286	14944			
11				5	-5.1049799131759	14944			
12				6	-5.0691699555679	14944			
13				7	-5.044891592723	14944			
14				8	-5.0315244615723	14944			
15				9	-5.0181173683788	14944			
16				10	-4.9668011211348	14944			
17				11	-4.9088471570678	14944			
18				12	-4.8562207686806	14944			
19				13	-4.8120991598731	14944			
20				14	-4.7739040728396	14944			
21				15	-4.7420846838351	14944			
22				16	-4.7130395372449	14944			
23				17	-4.6845119622478	14944			
24				18	-4.6512598021	14943			
25				19	-4.6059010361138	14942			
26				20	-4.5576600307074	14941			
27				21	-4.5089659197023	14940			
28				22	-4.4627637914451	14940			
29				23	-4.4199033592299	14940			
30				24	-4.3806473336851	14940			
31				25	-4.3438264729397	14940			
32				26	-4.3086882912853	14940			
33				27	-4.2765540303601	14940			
34				28	-4.2459716088783	14940			
35				29	-4.2178307453643	14940			
36				30	-4.190575075615	14940			
37				31	-4.1637247709992	14940			
38				32	-4.1374918780994	14940			
39				33	-4.1100291812289	14940			
40				34	-4.0827961369122	14940			
41				35	-4.0551640113178	14940			
42				36	-4.0265662144824	14940			
43				37	-3.9973239880987	14940			
44				38	-3.9687187808677	14940			
45				39	-3.9410926047463	14939			
46				40	-3.9151559979116	14939			
47				41	-3.890194204304	14937			
48				42	-3.8660270304996	14936			
49				43	-3.8425660821228	14934			
50				44	-3.8190040076758	14932			
51				45	-3.7954505133793	14931			
52				46	-3.7710239776345	14931			
53				47	-3.7461687790815	14931			
54				48	-3.7208506964285	14931			
55				49	-3.6941193122355	14930			
56				50	-3.6668011885369	14929			
57		4	1	0	-5.1693142597512	14946			
58				1	-5.0846414238358	14946			
59				2	-4.9858070500624	14946			

Sheet1

For Help, press F1 Row 1 Col 2 NUM

Fig. 36. Maximum Lyapunov Exponent Results.

The results of the Lyapunov Exponent are depicted on this window. The Estimated Lyapunov is “0.032414512233909”.

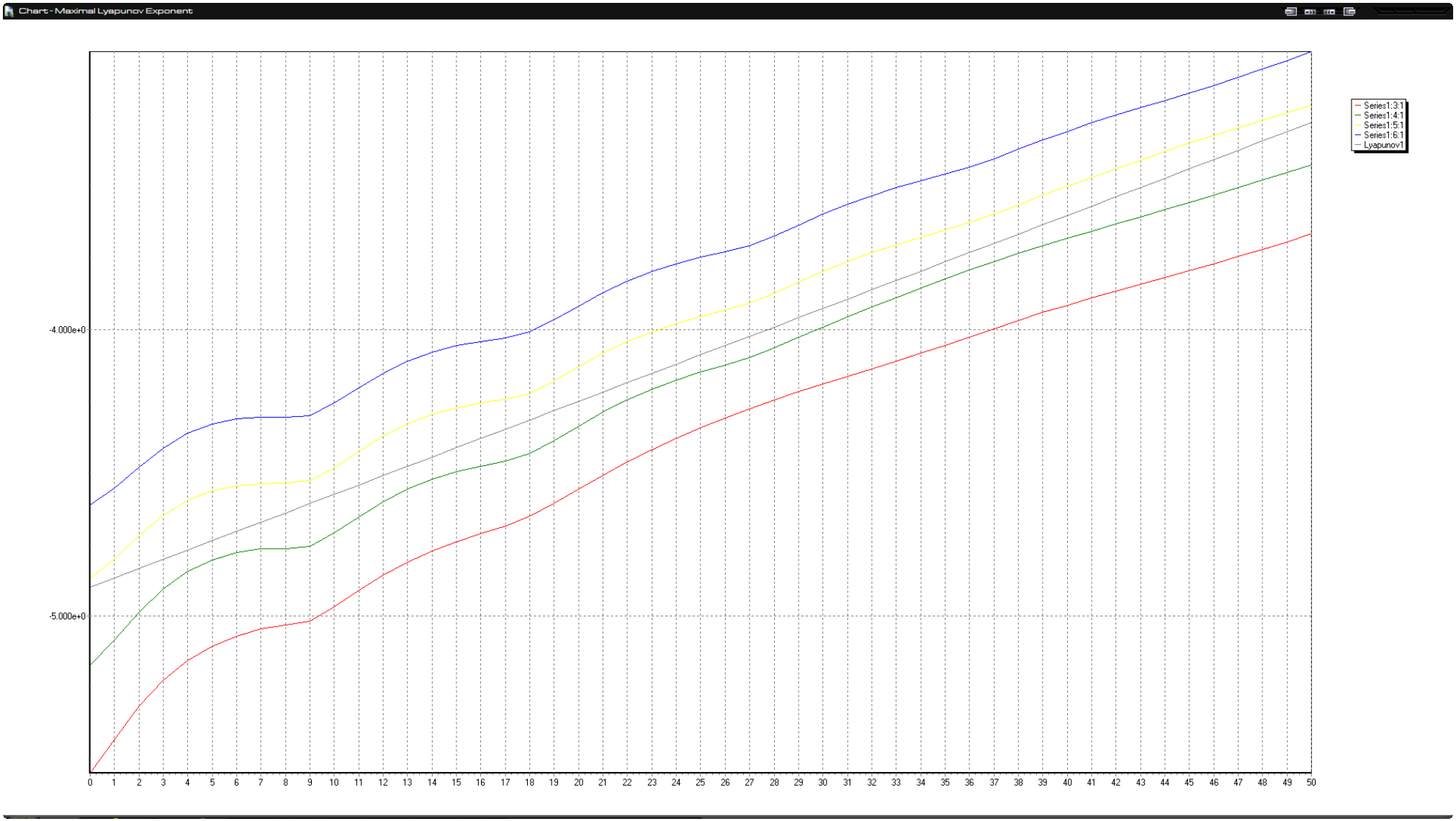

Fig. 37. Maximum Lyapunov Exponent Chart.

5.4. Analysis

5.4.1. Average Mutual Information Analysis

The analysis began with the Average Mutual Information Analysis configured as shown below:

- Series A: Valid Input Range [A1:A15041].
- Series B: Valid Input Range [A1:A15041].
- Max Delay: 100

The acquired results are the following:

- First Minimum: 9
- Absolute Minimum: 96

The system's Time Delay is 9.

The Average Mutual Information is used to determine the system's delay. This method was proposed as a method for finding the system's delay by Andrew M. Fraser and Harry L Swinney in the article "Independent coordinates for strange attractors from mutual information," Phys. Rev. A 33 (1986) 1134-1140). The idea of delay coordinates is that if you can observe a variable from a system X(t) and you want to reconstruct a higher dimensional attractor, you can consider X(t),X(t+T),…X(T+nT) to produce a (n+1)-dimensional coordinate which can be plotted. It is imperative that an appropriate value for T be chosen for too short a delay will consequently make X(t) very similar to X(t+T), ergo, all of the data will be near the line X(t)=X(t+T) when plotted. Too long a day will virtually render the effort fruitless as no useful information can be extrapolated from the plot. An appropriate choice for T is one that given a measurement of X(t), on average, many bits can be predicted about X(t+T).

5.4.2. Covariance and Correlation

The analysis continued with the Covariance and Correlation configured as shown below:

- Series A: Valid Input Range [A1:A15041].

- Series B: Valid Input Range [A1:A15041].
- Max Lag: 100

The acquired results are the following:

- Theiler Window: 82

The first nullification of the autocorrelation function, known Theiler Window, is the time that the Neighbours are spatially divergent.
If there is no nullification of the autocorrelation function then the absolute minimum of the Average Mutual Information is to be used as Theiler's Window.
It is essential that we have no false neighbours. If we imagine an ellipse and two anti-diametric points are selected to be projected to a line then they will be side by side, which is indeed false. If I look at the ellipse or circle from 1 dimension then I'll see the points as being neighbours, however, if I look at the points from a 2 or higher dimensional space then it becomes clear that they are not.

5.4.3. False Nearest Neighbours

I, then, proceeded to examine the False Nearest Neighbours Statistics configured as shown below:

- Input Range: Valid Input Range [A1:A15041].
- Time Delay: First Minimum [9]
- Exclusion Window: Theiler Window [82]

The acquired results are the following:

- Variables: 4
- The system's dimension is four.

This system is described by 4 variables/equations because in the False Nearest Neighbours graph it becomes apparent that, although there is no nullification, the line starts to become horizontal on the value 4.
The embedding dimension is the phase-space's number of dimensions *m* that are required for the proper embedding of the attractor, provided that an attractor exists. The

attractor's dimension must always be smaller than or equal to the embedding dimension.

5.4.4. Generalised Dimensions

The Generalised Dimensions analysis of Auguri is configured as shown below:

i. Input
   - Input Range: Valid Input Range [A1:A15041].

ii. Order and Embedding
   - Min. Dimensions: 1
   - Dimension Steps: 10 (anything greater than 10 is considered complex, therefore 10 is the maximum chaotic value)
   - Time Delay: First Minimum [9]

iii. Distance and Metrics
   - Minimum Epsilon: 0.01
   - Maximum Epsilon: 0.9

iv. Reference Points:
   - Exclusion Window: Theiler Window [82]

v. Plot: X axis:
   - Left Minimum: 0
   - Left Maximum: 10

The acquired results are the following:

- Fractal Dimension: 1.8254612489908

The dimension steps value is 10 because anything above 10 is not considered chaos. On [Fig. 33.] Generalised Dimensions - Series 1, Order 2: Dq vs. Embedding Dimension Chart it is revealed that Fractal Dimension FD = 1.8254612489908, which is not an integer and that is indicative of chaotic behaviour. Normal geometric figures have integer geometric dimensions. A line is a one-dimensional figure, a circle is a two-dimensional figure and a ball is a three-dimensional figure. Chaos takes place on non-

integer dimensions, hence, fractal dimension; just like the one revealed by the Generalised Dimensions Analysis of Auguri.

5.4.5. Lyapunov Exponent

The Lyapunov Exponent, named after Aleksandr Lyapunov, is an indicator of the rate of separation of infinitesimally close trajectories.

The Lyapunov Exponent analysis of Auguri is configured as shown below:

i. Input
   - Input Range: Valid Input Range [A1:A15041].

ii. Kantz
   - Min. Dimensions: 3
   - Max. Dimensions: 6
   - Time Delay: First Minimum [9]
   - Min. Neighbourhood: 10
   - Exclusion Window: Theiler Windows [82]

iii. Rosenstein
   - Min. Dimensions: 3
   - Max. Dimensions: 6
   - Time Delay: First Minimum [9]
   - Exclusion Window: Theiler Windows [82]

The acquired results are the following:

- Estimated Lyapunov Exponent: 0.032414512233909

The formula for figuring out the forecasting period is:

$$\frac{1}{L} \tag{16}$$

where L is the estimated Lyapunov.

The predictability of the system is:

$$\frac{1}{L} = \frac{1}{0.032414512233909} \cong 31 \text{ steps}$$

Ergo, the predictability of the system is 31 steps.

The Maximum Lyapunov Exponent calculated by Auguri provides useful insight as a means of a notion of predictability for the Circuit whose dynamic behaviour is being analysed.

A positive L indicates chaotic behaviour whilst a non-positive value indicates absence of chaos.

All the information points to Chua's circuit being a chaotic system. As a chaotic system, Chua's Circuit seems to be a valid candidate for communications encryption using chaos.

References

Books and Papers:

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2. Experimental Characterization of Chua's Circuit, Patrick Chang (December 2012), http://nldlab.gatech.edu/w/images/2/29/Patrick_Chang-final_paper.pdf
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Lectures:

1. Lecture Series on Chaos, Fractals and Dynamical Systems by Prof. S. Banerjee, Department of Electrical Engineering, IIT Kharagpur.
2. The Secret Life of Chaos by Prof. J. Al-Khalili, Department of Theoretical Physics, University of Surrey.